The welfare implications of the China Shock across U.S. States and Skill Groups

Juan Ignacio Jacoubian[†]

Boston University

January 29, 2026

Abstract

I build a dynamic quantitative spatial trade model with firm and worker heterogeneity, mobility frictions, and short-run unemployment to quantify the regional and distributional effects of the China Shock across U.S. states. The baseline model implies an average welfare gain (0.1%) but substantial heterogeneity across skill groups. Average welfare gains are 0.01% for unskilled workers—who lose in 17 of 50 states—versus 0.31% for skilled workers, who lose in only 7 states. The model predicts an average increase of 0.78% in the skill premium, with more exposed states experiencing larger increases. Finally, I find long-run gains in every state for both types of workers.

Keywords: China Shock; international trade; firm heterogeneity; wage inequality; skill premium; regional adjustment; mobility frictions; unemployment.

JEL codes: F12; F14; F16; D31; R12; R23.

---

[*]I am grateful to my advisor, Natalia Ramondo, for her guidance and support. I thank Masao Fukui and Joaquin Blaum, as well as Stefania Garetto, Yuhei Miyauchi, the Trade Advising Group, and participants in the Macro Seminar for helpful comments and discussions. I am also grateful to Cian Ruane and colleagues at the Central Bank of Ireland, and to my colleagues at Boston University, in particular Plinio Bicalho, Franco Maldonado, Martin Martinez and Alvaro Contreras. Some results are based on an analysis of strictly controlled Research Microdata Files provided by the Central Statistics Office (CSO). The CSO does not take any responsibility for the views expressed or the outputs generated from this research. Any remaining errors are my own.Email: jjacoubi@bu.edu.

1 Introduction

Since the early 2000s, the rapid integration of China into global trade has reshaped production and labor-market outcomes across advanced economies. A large empirical literature documents that this shift—commonly labeled the *China shock*—was driven by substantial productivity gains in China, which fueled a steep rise in U.S. import penetration (Autor et al., 2013). Over the same period, U.S. manufacturing experienced declining output and rising labor-market strain, particularly in regions more exposed to Chinese import competition, with especially pronounced effects among lower-skill workers.

Reduced-form evidence points to sizable impacts of Chinese import growth on employment, non-participation, and local economic conditions, but the aggregate welfare implications of this shock remain less settled. Measuring absolute effects involves combining reduced-form empirical results, which capture relative changes, with a general equilibrium model that translates them into aggregate outcomes. Recent structural trade papers such as Caliendo et al. (2019) and Rodríguez-Clare et al. (2022) have begun to address this question in environments with short-run labor-market rigidities. Similarly, Galle et al. (2023) and Burstein & Vogel (2017) analyze the impact of the China shock across different types of workers in a static and national-level framework. However, to the best of my knowledge, no existing framework quantifies the impact of the China shock on skilled and unskilled workers separately across regions within a unified structural model with short-run frictions. This gap is important: empirical work has documented heterogeneous responses for college versus non-college workers [1], and given the large share of low-educated workers in the U.S. labor force at the onset of the 2000s, before the China shock, accounting for this dimension is crucial for any meaningful assessment of its aggregate welfare effects.

In this paper, I develop a quantitative framework to characterize the distributional and welfare consequences of the China shock across skill groups and US states over time. I follow the vast empirical literature and show two empirical facts using ACS microdata that my quantitative model is going to match: (i) Employment losses and increases in non-

[1] See for example Autor et al. (2013) and Autor et al. (2025).

participation are markedly larger and substantially more persistent for non-college workers than for college-educated workers. (ii) Over the same period, the skilled-to-unskilled wage-bill ratio rises, consistent with an increase in the relative remuneration of high-skill workers following China's productivity-led export expansion. Then, I use confidential Irish data for the year 2000 to show the positive correlation between firm size, export status, and firms' skill worker composition. Highlighting the fact that less productive firms are more intensive in low-skilled workers relative to exporter firms.

Motivated by these patterns, I build on the structural trade frameworks of Rodríguez-Clare et al. (2022) (RUV) and Caliendo et al. (2019) (CDP), incorporating nominal wage rigidities as in Schmitt-Grohé & Uribe (2016). I introduce an additional margin of within-sector firm heterogeneity: each region and sector contains (i) a high-productivity Armington firm that sells both locally and abroad, and (ii) a low-productivity representative local firm that serves only the domestic market. While both firms share the same value-added share, Armington producers employ a higher skilled-to-unskilled ratio than purely local firms, a difference summarized by a single parameter $\kappa \in (0, 1)$, which disciplines relative skill intensity in a tractable and hat-algebra-compatible manner.

In this environment, the China shock affects skilled and unskilled workers through four channels: (i) import penetration interacts with the initial distribution of workers across sectors—manufacturing, especially computers and textiles, carries the largest predicted shock incidence; (ii) regional exposure varies substantially across U.S. states, driving heterogeneous local demand responses; and (iii) sectoral mobility elasticities and nominal wage rigidity differ by skill group so workers with lower mobility face sharper short-run employment losses and higher unemployment risk. (iv) Low-productivity and high-productivity firms may potentially face a different import competition effect, and so affect low-skill workers differently than high-skill workers. To preserve tractability and isolate worker-level incidence, migration elasticities and nominal-rigidity parameters vary by skill group but remain constant across regions. This richer structure allows the model to account for the heterogeneous reduced-form labor-market responses estimated in the data while delivering skill-specific welfare implications previously unquantified in a structural setting.

I calibrate the quantitative spatial general equilibrium model to match key features of U.S. and international data, including value added relative to sectoral revenues, input–output coefficients, consumption expenditure shares, and bilateral trade shares. The calibration also disciplines downward nominal wage rigidities and skill-specific sectoral mobility frictions. In the baseline calibration, I choose the nominal rigidity parameters and sectoral mobility elasticities so that the model reproduces the reduced-form effects in 2007 on unemployment and labor-force participation in response to Chinese import growth.[2]

Using the calibrated parameters, the baseline model predicts sizable heterogeneity in the regional and distributional effects of the China Shock. Unskilled workers experience welfare losses in 17 out of 50 U.S. states, with welfare changes ranging from -0.05% to 0.11%, with a weighted average of 0.01%. Skilled workers experience welfare declines in only 7 states, but the dispersion is larger: welfare changes range from -0.37% to 0.92%, with a weighted average of 0.31%. By the end of 2007, the model predicts an average increase of the skill premium of 0.78%.

2 Literature Review

This paper is related to several strands of the literature that analyze the impacts of trade shocks on different workers, regions, and countries. The closest benchmark frameworks are those developed by RUV and CDP, with additional insights drawn from within-sector firm heterogeneity mechanisms as in Burstein & Vogel (2017).

The seminal reduced-form evidence on U.S. import penetration from China is provided by Autor et al. (2013), Autor et al. (2016), and Autor et al. (2021). A comprehensive synthesis of this evidence and the evolution of the literature is documented in Autor et al. (2025), which compiles key stylized facts on import competition, employment, labor force non-participation, and the regional incidence of trade shocks across U.S. labor markets.

[2]Following the empirical conventions in CDP and RUV, I exclude Alaska and Hawaii when estimating these reduced-form moments because commuting-zone data are limited to the mainland U.S.

Within this broad literature, my paper extends the empirical evidence on the China Shock along two dimensions: its persistence over time and its heterogeneous effects across education groups. In particular, Autor et al. (2021) documented that the adverse effects of the China Shock on wages and labor force participation persist well beyond 2010 and remain statistically significant through 2019, nearly a decade after the peak of the shock. They also show that while migration responses are negligible for the overall working-age population (ages 18–65), positive migration effects emerge when focusing on younger cohorts (ages 25–39). Moreover, they provide evidence of substantial heterogeneity across local labor markets, showing that commuting zones with a lower share of high-skilled workers experience significantly larger and more persistent adverse effects. In addition, Autor et al. (2025) showed that most of the contribution to the reduction in employment in manufacture were workers with less than a college degree [3] and almost all of the negative earning effects were also coming from workers with less than a college degree [4]. Complementary evidence on distributional heterogeneity is provided by Chetverikov et al. (2016), who show that low-wage earners are disproportionately affected by the China Shock using quantile regression methods. Building on these insights, my empirical design combines the emphasis on heterogeneous exposure from this literature with the approach in Rodríguez-Clare et al. (2022), who estimate the long-run persistence of trade-induced effects on non-participation and unemployment. I show that both the magnitude and the persistence of these effects differ sharply between non-college and college-educated workers.

A large set of quantitative trade papers has extended this evidence to general-equilibrium structural models where the China shock is introduced as an increase in Chinese manufacturing firms' productivity, and welfare changes can be computed by combining price, production, employment, and demand responses. This class of models allows us to quantify welfare gains and losses generated by trade shocks across groups, rather than only the associated relative income effects, as emphasized in Caliendo et al. (2019) and Rodríguez-Clare et al. (2022). Within this strand, Caliendo et al. (2019) introduces labor mobility in the spirit of Artuç et al. (2010) and employment frictions across sectors and regions

[3] See Figure 6 panel C of the paper.
[4] See Figure 8, panels C and D.

and shows that, despite positive average welfare gains from trade liberalization, there is a reduction in the positive effects in comparison with models without these frictions. In the same line, RUV incorporates nominal rigidities in wages and separates the elasticity of mobility across sectors from the elasticity of migration across regions. Taking this into account, they show that a substantial share of U.S. regions—roughly 40%—experience welfare losses when calibrated to match the import competition effects of Autor et al. (2013). Adao et al. (2019) similarly analyzes heterogeneous regional incidence, identifying the importance of market-level import penetration for welfare outcomes.

In the context of Brazil, Dix-Carneiro & Kovak (2017) documents the effects of trade liberalization on employment dynamics across regions, showing the central role of imperfect interregional labor mobility, slow capital adjustment, and agglomeration economies in shaping the propagation of trade shocks on local labor demand. More recently, Dix-Carneiro et al. (2023) emphasizes the importance of avoiding balanced-trade restrictions in structural shock environments, showing that allowing endogenous shifts in trade deficits is crucial to correctly quantify the manufacturing and employment implications of the China shock. It is worth mentioning that all the papers mentioned above work in the context of a representative worker in each region.

A closely related literature analyzes distributional outcomes when workers differ by skill or occupation. Galle et al. (2023) combine an Eaton–Kortum model with a Roy structure to introduce labor heterogeneity across sectors and quantify welfare effects at the group level, finding that—even when average welfare rises—some labor groups experience losses up to five times larger than the average impact. The structural work of Burstein & Vogel (2017) introduces within-sector skill-biased productivity (SBP) driven by firm-level skill-intensity differences and shows that, relative to models without within-industry heterogeneity, trade liberalization amplifies wage inequality by raising the skill premium.

A growing literature has incorporated trade shocks into monetary and heterogeneous-agent structural environments, including Kim et al. (2023) and Waugh (2023). The latter introduces financial frictions on households, allowing for heterogeneous agents' asset

choices and disciplined welfare transmission through endogenous trade elasticities, but relies on stronger simplifications on the production side than state-of-the-art quantitative trade models. Finally, Kim et al. (2023) studies the impact of the China Shock at the national level for the United States, but develops a novel New Keynesian model, emphasizing the role of monetary policy and China's exchange rate peg during the period 2000-2007.

Finally, I analyze the relation between firm size, export status, and the white-collar share of workers across firms. Verhoogen (2008) shows that firms with higher domestic sales have a higher share of white-collar workers. Accetturo et al. (2013) shows that exports increase the share of skilled workers at the sectoral level. Bustos (2011) shows that increased export opportunities make the adoption of new technologies more profitable, thus raising aggregate demand for skilled workers. Similarly, Bernard & Jensen (1997) finds that exporters account for almost all of the increase in the wage gap between high- and low-skilled workers. I contribute to this literature by showing that for the year 2000, using Irish confidential data, there is a positive correlation between firm size and workers' distribution, and also between the export status of the firm and the share of white-collar workers.

In this context, my paper contributes to the literature in two ways. First, I develop a structural model that allows for calculating welfare for skilled and unskilled workers separately, while at the same time introducing firm heterogeneity without losing tractability of the model. I build most closely on Rodríguez-Clare et al. (2022) (RUV) and Caliendo et al. (2019) (CDP) by allowing for unemployment and mobility across sectors, but with frictions. Guided by the empirical evidence linking exporter size to skill intensity presented in section 4, I introduce a tractable within-sector firm heterogeneity mechanism featuring a purely local low-productivity representative firm (lp) alongside a productivity frontier Armington exporter (hp). Both firms have identical value-added shares, but local firms have a more unskilled-intensive production function. Unlike Burstein & Vogel (2017) —where a continuum of firms endogenously choose not to export—I impose lp firms exogenously to preserve tractability in a model that is already high-dimensional. This ad-hoc simplification retains the SBP transmission mechanism while allowing for a

consistent hat-algebra representation of the model [5]. Based on the empirical moments of Autor et al. (2013) and the empirical estimates of the relation between firm size, exporting, and workers' distribution, I expand the labor side to distinguish skilled and unskilled workers, allowing for differential nominal-wage rigidity (δ_g) and differential sectoral mobility elasticities. RUV and CDP allow mobility across sectors; I maintain this dimension but impose no movement across regions, consistent with micro evidence finding no significant effects on population migration (Autor et al., 2013). Under these richer worker margins, the model delivers skill-specific welfare effects that have not been quantified before.

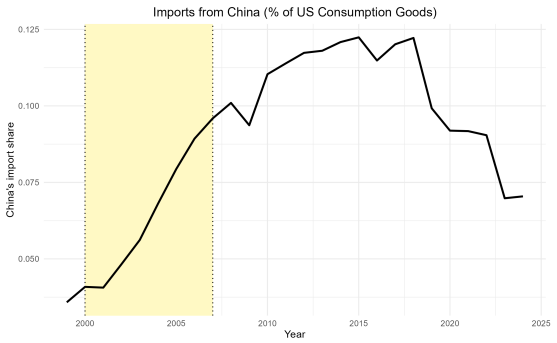
3 Trade exposure, skill level, and stylized facts

This section documents a set of stylized facts on trade exposure and labor market outcomes that motivate both the empirical analysis and the quantitative model developed later in the paper. Building on the extensive reduced-form literature on the China Shock, I first summarize the main aggregate patterns associated with the surge in Chinese import penetration and then extend this evidence by explicitly distinguishing between skilled and unskilled workers. The focus on skill heterogeneity is central, as prior work emphasizes large and persistent distributional effects of trade shocks, yet comparatively less is known about how these patterns differ across education groups over time and across regions.

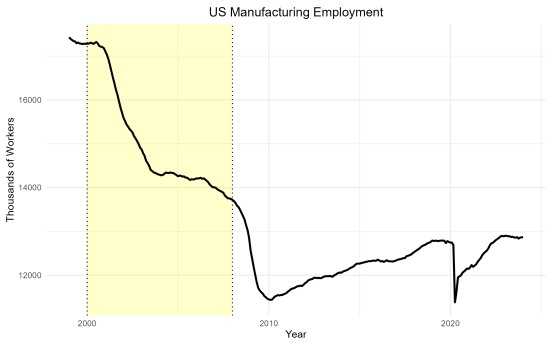
I begin by defining the China Shock and describing its evolution during the period under analysis. I then document national trends in import penetration, manufacturing employment, and the U.S. trade balance, highlighting the sharp increase in exposure to Chinese imports between 2000 and 2007 and its coincidence with a sustained decline in manufacturing employment. Next, I turn to regional variation in trade exposure across commuting zones and show how differences in exposure are systematically correlated with local labor market outcomes. Throughout the section, I emphasize how these relationships differ across skill groups, foreshadowing the heterogeneous and persistent responses that are central to the empirical results and the structural mechanisms explored in the model.

[5] It is true that introducing exit and entry in the model could increase the effect of the SBP mechanism.

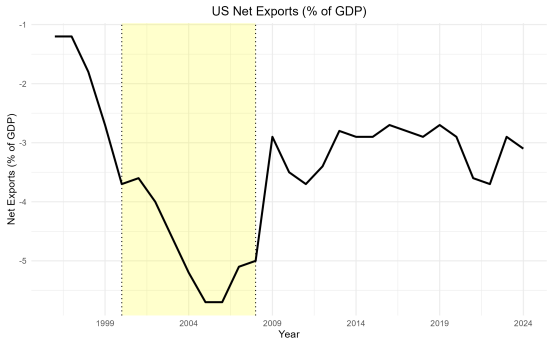
As in the existing literature, I define the China Shock as the large increase in U.S. manufacturing imports from China over the period 2000–2007:



(a) Chinese imports of goods



(b) U.S. manufacturing employment



(c) Net exports (% of GDP)

Figure 1: Three stylized facts on the China Shock period.

Sources: (a) BEA data. (b) BLS–FRED. (c) U.S. Census and BEA.

Figure 1 highlights three aggregate patterns that characterize the period under analysis: a sharp rise in the share of Chinese manufactured goods in U.S. consumption, a persistent decline in manufacturing employment, and a steady deterioration of the U.S. trade balance. These aggregate trends provide the backdrop for the regional analysis that follows and for the decomposition of effects by skill group.

3.1 Persistence Effect on Skilled and Unskilled Workers

To analyze the impact of the China Shock on various outcomes and its persistence over time, we combine the approaches of ADH and RUV and extend them to examine skilled and unskilled workers separately. Specifically, I employ the stacked first difference methodology: I consider the period from 1990 to 2000 and, for each horizon h (ranging from 7 to 21), the period from 2000 to $2000 + h$. In addition, I merge the ADH replication package data with the three-year-pooled ACS samples and estimate the following regression [6]:

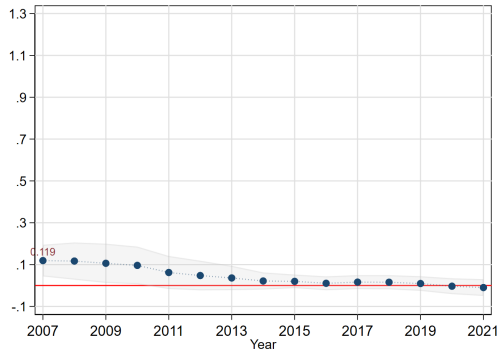
$$\Delta Y_{i,t+h} \;=\; \alpha_t \;+\; \beta_{1h}\, \Delta IP^{\text{CU}}_{i,\tau} \;+\; X'_{i,t}\, \beta_2 \;+\; \varepsilon_{i,t+h} \tag{1}$$

where $\Delta Y_{i,t+h}$ is the ten-year equivalent changes in the variable of interest and h goes from 7 to 21. As noted above, both the control variables and the trade-exposure measure are drawn from the ADH replication package, and I implement the same two-stage least squares strategy as in ADH and RUV. Figure 2 shows the results when analyzing the unemployment rate relative to the economically active population and the rate of individuals not in the labor force. We observe that among unskilled workers, the impact is larger in the initial period (2007) and remains more persistent. In particular, the effect on the non-labor-force rate (NILF) for unskilled workers stays statistically significant even in the final year of the sample. By contrast, for skilled workers, both NILF and the unemployment rate lose statistical significance rapidly. Meanwhile, the estimated impact on wages for each worker type is depicted in Figure 3. We can see a significant negative effect of import exposure on average annual earnings within CZs. However, we observe that the effect is even larger for unskilled workers and also slightly more persistent.

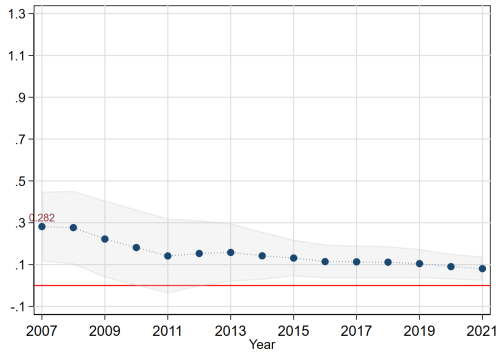
Finally, in Figure 4, I analyze the effect on the total income of skilled workers relative to that of unskilled workers. I define the relative payment as:

[6] I consider workers between 16 and 64 years old as ADH. Residents of institutional group quarters, such as prisons and psychiatric institutions are dropped along with unpaid family workers.

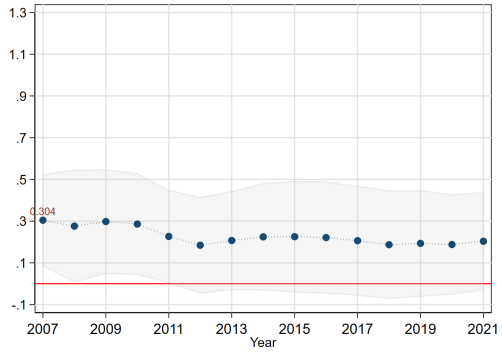
Figure 2: NILF and unemployment rate evolution



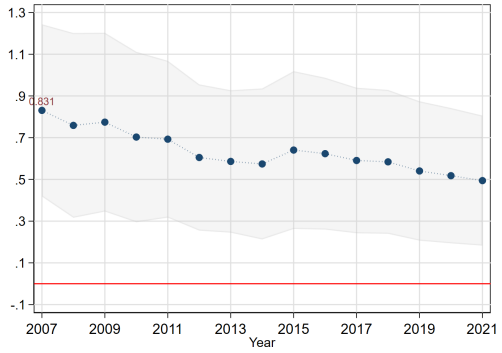
(a) Unemployment College



(b) Unemployment Non-College



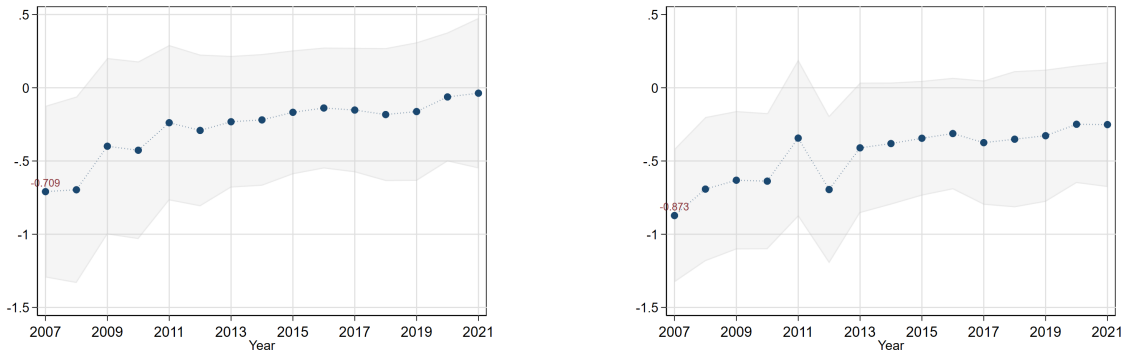
(c) NILF College



(d) NILF Non-College

The panels show, for each year on the horizontal axis, the two-stage least squares estimate of β_{1h} in equation (1), along with its 95% confidence interval. Each point comes from a separate IV regression that stacks the change in the outcome from 1990 to 2000 and from 2000 to that year.

Figure 3: Annual wage evolution

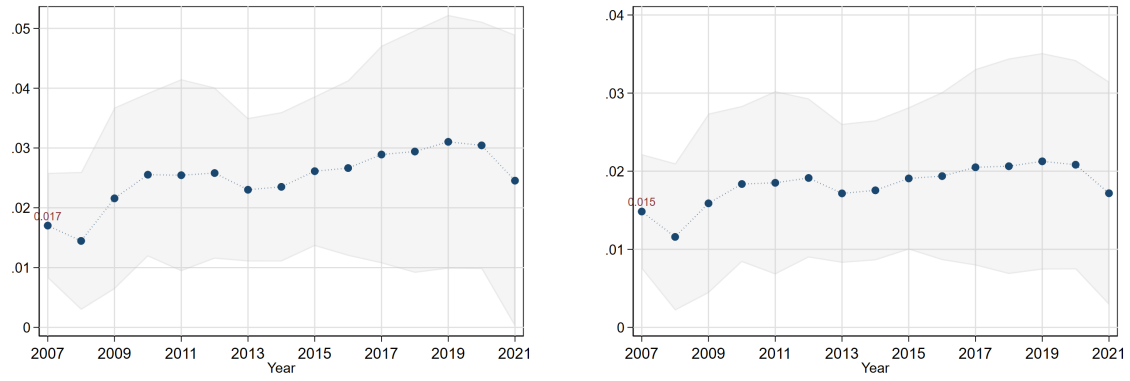


(a) College (b) Non-College

The panels show, for each year on the horizontal axis, the two-stage least squares estimate of β_{1h} in equation (1), along with its 95% confidence interval. In this case, the dependent variable is the average wage for skilled or unskilled workers in each CZ. Each point comes from a separate IV regression that stacks the decadal change in the outcome from 1990 to 2000 and the decadal change from 2000 to that year.

$$\frac{\sum_s w_{i,s,h,\text{Skilled},2000+h}\, L_{i,s,\text{Skilled},2000+h}}{\sum_s w_{i,s,\text{Unskilled},2000+h}\, L_{i,s,h,\text{Unskilled},2000+h}} \times \frac{\sum_s w_{i,s,h,\text{Unskilled},2000}\, L_{i,s,\text{Unskilled},2000}}{\sum_s w_{i,s,h,\text{Skilled},2000}\, L_{i,s,\text{Skilled},2000}}$$

where we sum over all sectors s for each period h. We observe a permanent rise of 1% to 3% for every 1000 US dollars of exposure to the China Shock in the relative income of skilled workers compared to unskilled workers in more exposed areas.



(a) Levels: skilled over unskilled wage payments

(b) Log change: Δ log skilled over unskilled wage payments

Figure 4: Skilled Workers over Unskilled Workers total labor payments

The panels show, for each year on the horizontal axis, the two-stage least squares estimate of β_{1h} in equation (1), along with its 95% confidence interval. Panel (a) uses the total wage payment of skilled workers relative to unskilled workers in levels. Panel (b) uses the log change of the same ratio.

3.2 Exposure measure and WIOD data

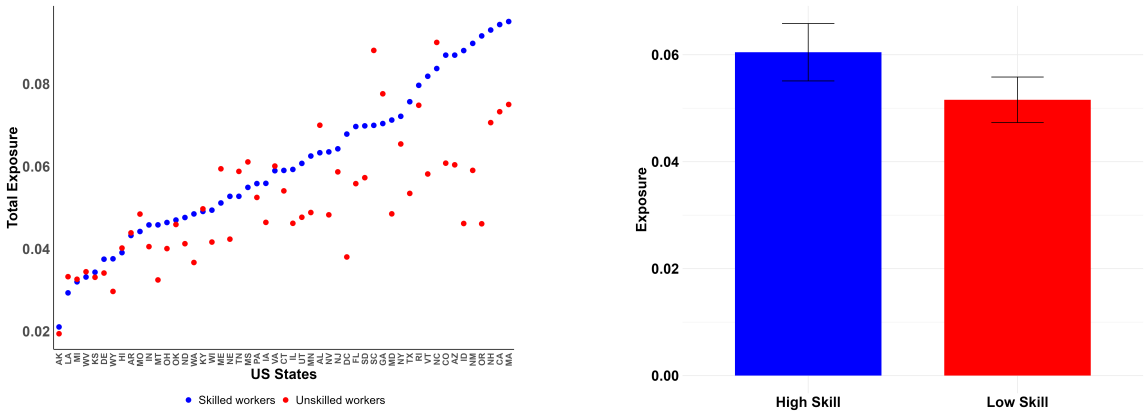
Before turning to the structural model, it is useful to connect the empirical variation at the commuting-zone level with the regional units used in the model. The model is defined at the state level, so I construct state-level measures of exposure to rising Chinese import competition by combining WIOD data at the national level with several US sources to map it at the state level. Unlike in the empirical analysis, and for comparison, I compute two measures—one for skilled workers and one for unskilled workers—given by [7]:

$$\text{Exposure}_{ij} = \sum_{s=1}^{S} \frac{L_{ij,s,2000}}{L_{ij,2000}} \frac{\Delta \hat{X}^{2007-2000}_{C,US,s}}{R_{US,s,2000}}.$$

Here, $L_{ij,s,2000}$ denotes the employment of skill group j in state i and sector s in 2000, and $L_{ij,2000} = \sum_s L_{ij,s,2000}$. The term $\Delta \hat{X}_{C,US,s}$ is the predicted change in U.S. imports

[7] For the structural approach, I use a unique exposure measure, following ADH. In addition, I scale up the exposure measure such that the mean across states equals the mean in ADH at the CZ level.

from China in sector s, following RUV and Autor et al. (2016).[8] I begin by examining exposure conditional on employment in manufacturing:



Exposure by state-skill level

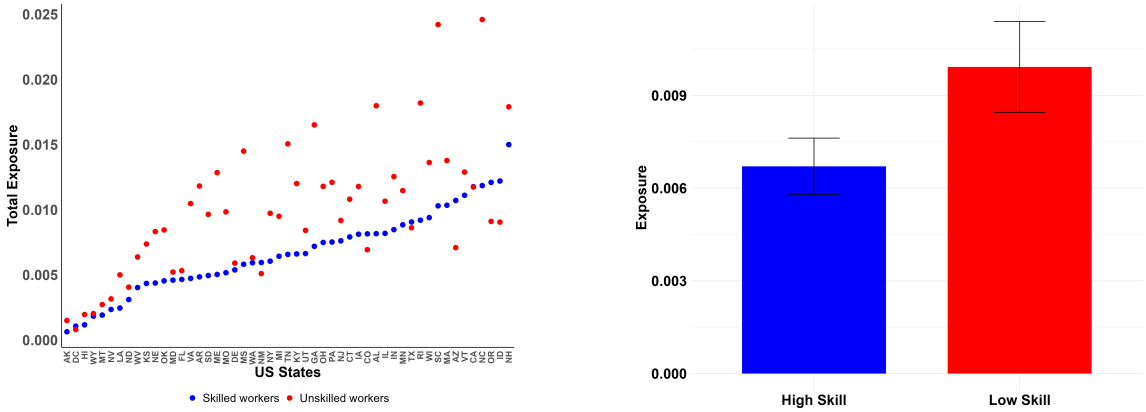
Exposure by skill level

Figure 5: Only Manufacturing

Skilled workers display higher exposure to the China shock within manufacturing. However, as shown in Figure (6), when all sectors are included, the ranking reverses. This shift reflects a simple composition effect. Skilled workers are more concentrated in service and agricultural industries, which saw relatively small increases in Chinese import penetration. Unskilled workers remain disproportionately employed in manufacturing sectors that absorbed most of the import shock. As a result, aggregate exposure is higher for unskilled workers.

These patterns underscore that exposure to rising Chinese import competition is shaped jointly by workers' skill composition and states' sectoral mix. This motivates the heterogeneous-worker, multi-sector structure of the model introduced in the next section. As mentioned before, another important channel to analyze the impact of the trade shock is the within-sector heterogeneity across firms. The next section shows this fact.

[8] Following ADH, the empirical specification uses labor rather than R in the denominator.



Exposure by state-skill level

Exposure by skill level

Figure 6: All Sectors

4 Firm size, export status and worker heterogeneity

A salient empirical regularity in the trade and inequality literature is that import-competition shocks transmit unevenly across worker types, and that these effects are often disciplined by the skill composition of employment within sectors. At the same time, a complementary plant- and firm-level literature shows that exporters and larger producers make more intensive use of high-skill labor. Motivated by this evidence, the model introduces an additional within-sector productivity margin that disciplines how import competition is allocated across worker groups through differential skill intensity in production.

To justify this separation empirically, I rely on confidential Irish administrative firm-level data provided for the period 1998–2001. Because the structural exercise uses the year 2000 as the baseline, I retain observations from that year whenever possible. When a firm appears multiple times in the sample, I keep only its 2000 observation to preserve a one-to-one cross-sectional mapping between firm characteristics and model targets.

I combine data from the Irish Census of Industrial Production (CIP) with custom data to determine the export status of each firm [9]. The sample contains 5,613 firms. From these

[9]This confidential dataset has been used widely in the literature. See, for example Fitzgerald & Haller

data, I compute the share of managers and technical workers relative to total employment (managers, clerical, and industrial workers) at the firm level, and I compute the white-collar share by including clerical workers [10]. Table 1 provides more details about the data.

Table 1: Summary Statistics by Firm Size (Gross Output Terciles)

	All	Bottom tercile	Middle tercile	Top tercile
Gross Output	17.8	0.30	1.20	51.9
	(175)	(0.16)	(0.48)	(300)
Share Managers/Technical	0.152	0.127	0.164	0.165
	(0.145)	(0.145)	(0.136)	(0.149)
Share White Collar	0.268	0.219	0.288	0.297
	(0.198)	(0.215)	(0.181)	(0.189)
N	5613	1871	1871	1871

To measure the interaction between firm size and skill composition, I estimate the following cross-sectional relationship:

$$\frac{h_i}{h_i + l_i} = \beta_0 + \beta_1 \log(\text{Gross Output}_i) + \text{Sector FE} + \epsilon_i,$$

where the sector fixed effect corresponds to 2-digit industry definitions under NACE Rev. 2. The dependent variable captures the skill intensity of a firm. I run the regression for the share of managers and the share of white-collar workers as defined above.

I consider two complementary estimation approaches. First, I estimate β_1 using OLS with robust standard errors. Under this specification, I obtain $\hat{\beta}_1 = 0.0027$ (0.001) for the share of manager and technical workers, and $\hat{\beta}_1 = 0.0082$ (0.001) for the share of white-collar workers[11], indicating a positive elasticity between firm output and skilled

(2014) and Fitzgerald et al. (2024). Some firms report themselves as exporters in the CIP dataset, but there are no exports in the customs data. I assume that they are not exporters for the regression.

[10] I defined white-collar workers as the sum of managers, technical and clerical workers, and use it as a proxy of skilled workers and industrial workers as non-skilled, respectively

[11] RSE in parentheses.

employment share. Second, I estimate a fractional regression model following Papke & Wooldridge (2008), which returns a point estimate $\hat{\beta}_1$ of 0.0207 (0.008) and 0.0428 (0.007) with an average marginal effect of 0.0026 and 0.0081, respectively. These semi-elasticities are aligned with the cross-sectional premia documented in Mexican manufacturing by Verhoogen (2008), who finds a point estimate of 0.01.[12]

As expected, and in line with structural models (Melitz, 2003), Table 2 provides a short description of the relation between firm size and export status. It is worth mentioning that the relative size of exporter firms and non-exporter firms is almost equal to Bernard et al. (2003), who reports that exporter firms are 5.6 times bigger than non-exporter firms for US firms.

Table 2: Summary Statistics by Export Status

	All	Non-exporters	Exporters
Gross Output	17.8	6.14	32.8
	(175)	(52.3)	(256)
Share Managers	0.152	0.14	0.167
	(0.145)	(0.141)	(0.148)
Share White-collar	0.268	0.248	0.292
	(0.145)	(0.20)	(0.195)
N	5613	3155	2458

I continue by estimating the impact of firm size (gross output) on export status. To do this, I run two complementary regressions, both controlling for sector FE as before. Starting with an OLS estimation, I get a point estimate of 0.068 (0.003) for the effect of (log) gross output on the binary variable equal to 1 if the firm exports and 0 otherwise. In addition, I run a logit regression, and the average marginal effect is 0.078 (0.02).

Finally, I estimate the correlation between the export status of the firm and the share of white-collar workers, controlling for sectoral fixed effects. As before, I run the regression for the share of managers/technicals and for the share of white-collar workers as defined

[12] See Table II, last column.

above. I find a coefficient of 0.012 (0.004) and 0.032 (0.005) for the share of managers and white-collar workers, respectively.

Overall, these estimates support the model's parsimonious productivity split governed by $\kappa \in (0, 1)$, capturing the empirically higher reliance on unskilled labor compensation among low-productivity local firms relative to exporters or standard Armington firms. The next section outlines the model and solution methodology.

5 Baseline Model

Building on Artuç et al. (2010), CDP, and RUV, I develop a dynamic, multi-sector quantitative trade model with input–output linkages in which forward-looking workers choose the sector in which to work within each region. To match the empirical patterns documented before separately for skilled and unskilled workers, I extend CDP and RUV along several dimensions: (i) wage rigidity (and unemployment) for each skill group; (ii) a labor-supply structure that allows different households to have distinct elasticities for switching across sectors (including non-employment), with these elasticities differing by skill [13]; and (iii) within-sector skill-biased productivity (SBP) in the spirit of Burstein & Vogel (2017), which generates systematic differences in the relative demand for skilled and unskilled labor across sectors and regions.

Basic Assumptions

Time is discrete, and there is no aggregate uncertainty. The world consists of multiple regions: M U.S. regions and $N-M$ foreign regions, following RUV and CDP. Each region

[13] Autor et al. (2025) showed that the reduction in the manufacturing to population ratio comes mainly from a change in the entry decision of new workers, and not from a real swith from manufacturing to non-manufacturing workers. However, trade pressure does increase the probability that incumbent workers transition to non-employment. I take the simplifying assumption of no new workers and assume that all the effects come from mobility across sectors of current workers.

contains S market sectors and one home-production sector (sector 0). In every region j, sector s, and period t, there is a mass of skilled workers $N_{jHs,t}$ and unskilled workers $N_{jLs,t}$. Households consume a composite good $C_{j,t}$ at price $P_{j,t}$ and allocate all income to consumption. Aggregate consumption is Cobb–Douglas across the S market sectors with expenditure shares $\phi_{j,s}$. Consumption in each sector is a CES aggregator over varieties produced in the N regions, with elasticity of substitution σ_s. All prices and wages are expressed in U.S. dollars. Firms pay a sector- and skill-specific wage $W_{jgs,t}$, while workers receive a wage $w_{jgs,t}$, which may differ due to wage rigidity and unemployment. Workers are forward-looking with perfect foresight and remain in their region. They select the sector in which to work—potentially including non-employment—while facing sectoral mobility costs that govern reallocation dynamics.

On the production side, each sector in every region features two types of firms: (i) low-productivity local firms, which sell only domestically, and (ii) standard Armington firms, which produce tradable varieties sold both locally and internationally. This structure generates within-sector productivity differences consistent with the skill-biased mechanism discussed above and allows trade shocks to propagate differentially to skilled and unskilled workers.

Firm Side

Each region i and sector s is populated by two types of producers: (i) local firms, which serve only the domestic market, and (ii) Armington firms, which sell both domestically and internationally. Both types share the same production technology but differ systematically in their skill intensity. A firm of type $p \in \{\text{local}, \text{Armington}\}$ operating in region i, sector s, and time t produces output according to

$$y_{is}^{p} = T_{is} \left(\left[\mu_{is,p}^{1/\rho} L_{iHs}^{(\rho-1)/\rho} + (1-\mu_{is,p})^{1/\rho} L_{iLs}^{(\rho-1)/\rho} \right]^{\frac{\rho}{\rho-1}} \right)^{\alpha_{is}} \prod_{k} x_{iks}^{\alpha_{iks}},$$

where T_{is} is sectoral productivity, α_{is} is the value-added share, α_{iks} are intermediate-input shares, and ρ governs the elasticity of substitution between skilled and unskilled labor at the firm level [14]. The parameter $\mu_{is,p}$ determines the relative weight of skilled labor in the composite labor input.

To capture the empirical fact that exporters and larger firms employ relatively more skilled workers than purely local firms, I allow the skill weight to vary by firm type and assume

$$\mu_{is,\text{local}} = \kappa\, \mu_{is,\text{Armington}}, \qquad 0 < \kappa < 1,$$

so that local firms place a lower weight on skilled labor and therefore employ a more unskilled-intensive workforce. Cost minimization implies that, for any firm type p, the relative demand for skilled and unskilled labor satisfies

$$\frac{L^p_{iHs}}{L^p_{iLs}} = \frac{\mu_{is,p}}{1-\mu_{is,p}} \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho},$$

which yields the cost-share ratio

$$\frac{W_{iHs}L^p_{iHs}}{W_{iLs}L^p_{iLs}} = \frac{\mu_{is,p}}{1-\mu_{is,p}} \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho-1}.$$

Since firms of each type face the same wages (W_{iHs}, W_{iLs}), differences in labor composition are governed entirely by the parameter $\mu_{is,p}$, and therefore by κ. It is convenient to summarize the labor composite with the unit cost index

$$W^p_{is} = \left[\mu_{is,p} W^{1-\rho}_{iHs} + (1-\mu_{is,p}) W^{1-\rho}_{iLs}\right]^{\frac{1}{1-\rho}},$$

so that a firm of type p implicitly chooses its skilled and unskilled inputs by choosing total labor L^p_{is} at price W^p_{is}. This formulation generates within-sector productivity differences consistent with the skill-biased mechanism discussed above and plays a key role in shaping how trade shocks transmit to different worker groups.

[14] In Appendix E I show that ρ is not equal to the elasticity of substitution. However, for empirically relevant values of κ, it is very close to it.

Sectoral Prices

Within each sector s, consumers in region j aggregate varieties from all regions using a CES aggregator with elasticity of substitution σ_s. This structure delivers standard demand functions with constant trade elasticities and implies that firms face a constant markup over marginal cost. The parameter σ_s governs both the degree of substitution across varieties within sector s and the sensitivity of trade flows to relative prices.

Given the cost structure described above, the optimal price set by an Armington firm located in country i and selling to country j in sector s is

$$p_{ijs} = \frac{\tau_{ijs}}{T_{is}} W_{is}^{\alpha_{is}} \prod_k P_{ik}^{\alpha_{iks}}, \tag{2}$$

where τ_{ijs} denotes iceberg trade costs from i to j, T_{is} is a productivity shifter, W_{is} is the sectoral labor cost index, and $\{P_{ik}\}$ are sectoral input prices.

Low-productivity local firms sell only in their domestic market and therefore do not incur trade costs. Their optimal price in sector s is given by

$$p_{iis}^{\text{lp}} = \frac{1}{T_{is}^{\text{lp}}} \left(W_{is}^{\text{lp}}\right)^{\alpha_{is}} \prod_k P_{ik}^{\alpha_{iks}}, \tag{3}$$

where T_{is}^{lp} and W_{is}^{lp} denote the productivity and labor cost index of local firms. The sectoral price index in market j for sector s then aggregates all available varieties under CES preferences. Relative to the standard Armington model, the key difference is that each sector now includes, in addition to the Armington varieties from all regions, an extra local variety supplied by low-productivity firms. The resulting price index is

$$P_{js}^{1-\sigma_s} = \sum_{i=1}^{N} p_{ijs}^{1-\sigma_s} + \left(p_{jjs}^{\text{lp}}\right)^{1-\sigma_s},$$

so that the presence of local low-productivity firms expands the set of varieties available

in the domestic market and affects the sensitivity of sectoral prices to trade shocks.

Goods Market

Goods market clearing equates the value of production of each variety (i, s) with global demand across all destinations. Let R_{is} denote revenues of region i in sector s. Under the CES structure introduced above, bilateral demand for the Armington variety produced in (i, s) takes the standard gravity form, so that

$$R_{is} = \sum_{j} X_{ijs}.$$

Using sectoral final expenditure shares ϕ_{js}, final consumption $C_j P_j$, and intermediate-input coefficients $\{\alpha_{jsk}\}$, the goods-market condition becomes

$$R_{i,s} = \sum_{j \notin US} \lambda_{ij,s}\, \phi_{j,s} C_j P_j + \phi_{US,s} \sum_{j \in US} \lambda_{ij,s} C_j P_j + \sum_{j=1}^{N} \lambda_{ij,s} \sum_{k=1}^{K} \alpha_{jsk} R_{jk}. \tag{4}$$

Household expenditure in region i satisfies

$$P_i C_i = \sum_{s} w_{iHs} L_{iHs} + \sum_{s} w_{iLs} L_{iLs} + \Delta_i, \tag{5}$$

where Δ_i denotes net foreign transfers (trade deficits or surpluses) that close the expenditure identity, taken as exogenous in the spirit of Dekle et al. (2007).

Local vs. Armington Shares. Because each sector includes two distinct domestic varieties—one produced by the low-productivity local firm and one by the Armington firm—the CES aggregator must distinguish between their market shares. Let P_{is} denote

the sectoral price index in region i. Demand for the local low-productivity variety is

$$\lambda_{iis}^{lp} = \frac{(p_{iis}^{lp})^{1-\sigma_s}}{P_{is}^{1-\sigma_s}},$$

while demand for the domestic Armington variety is

$$\lambda_{iis}^{hp} = \frac{(p_{iis})^{1-\sigma_s}}{P_{is}^{1-\sigma_s}}.$$

These two terms replace the single home share in the standard Armington model and reflect the presence of two domestic producers with different productivities and skill intensities.

Labor Market

Labor demand must account for the fact that total revenue R_{is} is generated by two types of producers. For foreign destinations $(i \neq j)$, all sales are made by the Armington firm. However, for domestic sales $(i = j)$, revenues must be split between the Armington firm and the low-productivity local firm. Thus, R_{is} represents total sectoral revenue in region i, combining domestic and export sales from Armington firms and domestic sales from local firms. Using the domestic CES shares defined above, revenues for each type of firm in region i, sector s, and period t are

$$R_{ist}^{lp} = \lambda_{iist}^{lp} E_{ist},$$

$$R_{ist}^{hp} = \sum_{j} \lambda_{ijst} E_{jst} - R_{ist}^{lp},$$

where $E_{jst} = \phi_{js} C_j P_j + \sum_k \alpha_{jsk} R_{jk}$ is total sectoral expenditure in (j, s). Unskilled labor demand satisfies

$$W_{iLs} L_{iLs} = (1 - \mu_{is}^{hp}) \left(\frac{W_{is}^{hp}}{W_{iLs}} \right)^{\rho - 1} \alpha_{is} \, R_{is}^{hp} + (1 - \mu_{is}^{lp}) \left(\frac{W_{is}^{lp}}{W_{iLs}} \right)^{\rho - 1} \alpha_{is} \, R_{is}^{lp}, \quad (6)$$

and skilled labor demand satisfies

$$W_{iHs} L_{iHs} = \mu_{is}^{hp} \left(\frac{W_{is}^{hp}}{W_{iHs}} \right)^{\rho - 1} \alpha_{is} \, R_{is}^{hp} + \mu_{is}^{lp} \left(\frac{W_{is}^{lp}}{W_{iHs}} \right)^{\rho - 1} \alpha_{is} \, R_{is}^{lp}. \quad (7)$$

These expressions make explicit how differences in skill intensity across firm types—captured by μ_{is}^{hp} and $\mu_{is}^{lp} = \kappa \mu_{is}^{hp}$—translate sectoral revenues into demand for skilled and unskilled labor.

Nominal Rigidity

I introduce downward nominal wage rigidity following Schmitt-Grohé & Uribe (2016). Wages are set in local currency units (LCU), and for each skill group g and sector s, nominal wages must satisfy

$$W_{igs}^{LCU} \geq \delta_g \, W_{igs,-1}^{LCU}, \qquad \delta_g \geq 0.$$

Since the model is expressed in U.S. dollars, let $e_{i,US,t}$ denote the exchange rate between region i's currency and the U.S. dollar, expressed as region-i currency for each unit of US dollars. Nominal wages in dollars are then

$$W_{igs} = \frac{W_{igs}^{LCU}}{e_{i,US,t}}.$$

Rewriting the rigidity constraint in dollars gives

$$W_{igs} \geq \underbrace{\frac{e_{i,US,t}}{e_{i,US,t-1}}\,\delta_g}_{\equiv \delta_{ig}}\, W_{igs,-1}. \tag{8}$$

Under flexible exchange rates, I assume $\delta_{ig} = 0$ for all foreign regions ($i \neq US$) and $\delta_{ig} = 1$ for U.S. regions in my baseline specification, similar to Rodríguez-Clare et al. (2022). Thus, downward nominal wage rigidity binds only in the United States.

Labor supply in each region i, sector s, and skill group g is given exogenously by N_{igs}, while firms demand L_{igs} units of labor. The labor-market condition is therefore

$$L_{igs} \leq N_{igs}. \tag{9}$$

Downward rigidity implies the following complementary-slackness condition:

$$(N_{igs} - L_{igs})\ (W_{igs} - \delta_{ig} W_{igs,-1}) = 0. \tag{10}$$

Temporary Equilibrium

Following CDP, a temporary equilibrium characterizes the static allocation in a given period, taking as given the distribution of workers across regions, sectors, and skill groups. All other variables in the static block—sectoral prices, revenue, expenditure shares, and trade flows— are algebraic functions of wages and employment.

Definition 1. *Given the stock of workers $\{N_{igs}\}$ across regions i, sectors s, and skill groups $g \in \{H, L\}$, a temporary equilibrium is a vector of wages and employment levels*

$$\{W_{igs},\ L_{igs}\}_{i,s,g}$$

such that:

1. ***Optimal Pricing.*** *Firms set prices according to the optimal-pricing condition equation* (2) *and* (3) *implied by wages and input costs.*

2. ***Goods-market-consistent revenues.*** *The sectoral revenues implied by* $\{W_{igs}, L_{igs}\}$ *satisfy the goods-market condition* (4) *and* (5).

3. ***Labor demand.*** *Skilled and unskilled labor demand satisfy the labor-demand conditions for both firm types (equations* (6)–(7)*).*

4. ***Nominal wage rigidity.*** *Wages satisfy the downward nominal rigidity constraint* (8).

5. ***Labor-supply feasibility.*** *Employment satisfies* (9).

6. ***Complementary slackness.*** *Wage rigidity and employment satisfy* (10).

All remaining variables—prices, price indices, revenues, trade flows, and expenditure shares—are obtained as algebraic functions of the equilibrium wages and employment levels.

Migration Choice

At the beginning of each period, workers take as given the prevailing conditions in all labor markets and the realization of their idiosyncratic preference shocks. Workers who are matched to a sector supply labor and earn the corresponding wage, while those not employed receive home-production utility. After receiving their flow utility, all workers—regardless of employment status—decide whether to remain in their current sector or switch to another one within the same region.

Let $v_{j,g,s,t}$ denote the lifetime utility of a worker with skill type $g \in \{H, L\}$ located in region j, employed in sector s, and making a choice at time t. The value function satisfies

$$v_{j,g,s,t} = \underbrace{U_{j,g,s,t}}_{\text{Flow utility}} + \max_{k \in \{0,\ldots,S\}} \left\{ \beta \, E[v_{j,g,k,t+1}] - \kappa_{j,s,k,g} + \nu_g \, \epsilon_{j,g,k,t} \right\},$$

where $\kappa_{j,s,k,g}$ is the mobility cost of switching from sector s to sector k, and $\epsilon_{j,g,k,t}$ are i.i.d. Type-I extreme value shocks with distribution

$$F(\epsilon) = \exp\left(-\sum_{k=0}^{S} \exp\{-\epsilon_{j,g,k,t}\} \right).$$

Let $V_{j,g,s,t} = E[v_{j,g,s,t}]$ denote the ex-ante value function. Standard results for nested logit imply

$$V_{j,g,s,t} = U_{j,g,s,t} + \ln\left(\sum_{k=0}^{S} \exp(\beta V_{j,g,k,t+1} - \kappa_{j,s,k,g})^{1/\nu_g} \right)^{\nu_g} + \gamma\nu_g, \tag{11}$$

where ν_g governs the elasticity of sectoral mobility and γ is Euler's constant. The corresponding migration probabilities—i.e., the fraction of workers transitioning from sector s to sector k—are

$$\mu^g_{j,s,k,t} = \frac{\exp\left(\beta V_{j,g,k,t+1} - \kappa_{j,s,k,g}\right)^{1/\nu_g}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,g,h,t+1} - \kappa_{j,s,h,g}\right)^{1/\nu_g}}. \tag{12}$$

Given these mobility flows, worker stocks evolve according to

$$N_{j,g,k,t} = \sum_{s=0}^{S} N_{j,g,s,t-1}\ \mu^g_{j,s,k,t-1}. \tag{13}$$

For tractability, I assume log utility. In addition, each worker with skill type g in region i and sector s receives labor income equal to the average wage in that labor market:

$$w_{i,g,s,t} = \frac{W_{i,g,s,t}\, L_{i,g,s,t}}{N_{i,g,s,t}}.$$

Nominal Anchor

With downward nominal wage rigidity, the level of nominal variables matters for the extent to which the constraint binds. Following RUV, I impose a simple nominal anchor

by assuming that world nominal GDP in U.S. dollars remains constant over time. This prevents a purely nominal drift in which all wages and prices grow fast enough that the rigidity constraint never binds and unemployment fails to materialize. Under this assumption, the parameter δ_g captures all nominal rigidities in the model. If the exogenous growth rate of world nominal GDP were different from zero, changing the level of world nominal GDP would rescale δ_g up or down without altering real allocations. For this reason, I normalize the path of world nominal GDP to be constant and interpret the calibrated values of δ_g as summarizing the effective degree of downward nominal wage rigidity, in line with the approach in RUV.

Sequential Competitive Equilibrium

Following the notation and structure in CDP, I now define a sequential competitive equilibrium for the dynamic economy. The state of the economy at the beginning of period t is summarized by the distribution of workers across regions, sectors, and skill groups, $\{N_{j,g,s,t}\}$, and by past wages $\{W_{i,g,s,t-1}\}$, which determine the extent to which nominal rigidities may bind.

Given initial conditions $\{W_{i,g,s,0}, L_{i,g,s,0}, N_{j,g,s,0}\}$ and sequences of exogenous fundamentals (trade costs, productivities, and net transfers), a *sequential competitive equilibrium* is a collection of sequences

$$\{W_{i,g,s,t},\ L_{i,g,s,t},\ N_{j,g,s,t},\ V_{j,g,s,t},\ \mu^{g}_{j,s,k,t}\}_{t=0}^{\infty}$$

such that, for all $t \geq 0$:

1. **Household problem.** The value functions $\{V_{j,g,s,t}\}$ satisfy the dynamic programming equation given migration costs and future continuation values, as in equation (11).

2. **Migration choices.** Sectoral mobility decisions are consistent with optimal choice probabilities, i.e. migration flows $\mu^{g}_{j,s,k,t}$ satisfy equation (12).

3. **Law of motion for worker stocks.** The distribution of workers across sectors and skill groups evolves according to the flow equation (13).

4. **Static block (temporary equilibrium).** For each period t, given the worker distribution $\{N_{j,g,s,t}\}$ and past wages, the vectors of wages and employment $\{W_{i,g,s,t}, L_{i,g,s,t}\}$ constitute a temporary equilibrium as defined above: they are consistent with goods-market clearing, firm optimality, labor demand, nominal wage rigidity, and the labor supply and complementary slackness conditions.

6 Hat Algebra

Since the economy begins from a stationary equilibrium, all variables are constant prior to the China shock. This allows me to use dynamic hat algebra, expressing every endogenous variable x_t in terms of its gross rate of change $\dot{x}_t \equiv x_t/x_{t-1}$ in the economy where the shock is realized. Throughout, period $t = 1$ denotes the pre-shock steady state, so $\dot{x}_1 = 1$ for all variables given that the shock occurs in period 2. I therefore express all counterfactual paths directly in terms of $\dot{x}_t$, rather than in terms of their $\hat{x}_t$ notation.[15] I now restate the full model in dynamic hat-algebra form. All objects indexed by $t-1$ are stationary steady-state variables. Given that low-productivity local firms are the new feature of the model, I denote their variables with a superscript lp. For the standard Armington firms, I keep the notation without a superscript (or with hp when needed for clarity).

Prices.

The labor cost of a low-productivity local firm is

$$\dot{W}_{is,lp,t}^{1-\rho} = \frac{\kappa\mu_{is}W_{iHs,t-1}^{1-\rho}}{\kappa\mu_{is}W_{iHs,t-1}^{1-\rho} + (1-\kappa\mu_{is})W_{iLs,t-1}^{1-\rho}}\dot{W}_{iHs,t}^{1-\rho} + \frac{(1-\kappa\mu_{is})W_{iLs,t-1}^{1-\rho}}{\kappa\mu_{is}W_{iHs,t-1}^{1-\rho} + (1-\kappa\mu_{is})W_{iLs,t-1}^{1-\rho}}\dot{W}_{iLs,t}^{1-\rho},$$

[15] CDP and RUV define $\hat{y}_t = \dot{y}'_t/\dot{y}_t$. In a stationary baseline, $\dot{y}_t = 1$ for all t, so $\hat{y}_t = \dot{y}'_t$. I therefore work directly with $\dot{y}_t$. This notation indeed generates problems in the first period of the shock, but I take it into careful consideration while solving the model.

while the labor cost of a traditional Armington firm is

$$\dot{W}_{is,t}^{1-\rho} = \frac{\mu_{is} W_{iHs,t-1}^{1-\rho}}{\mu_{is} W_{iHs,t-1}^{1-\rho} + (1-\mu_{is}) W_{iLs,t-1}^{1-\rho}} \dot{W}_{iHs,t}^{1-\rho} + \frac{(1-\mu_{is}) W_{iLs,t-1}^{1-\rho}}{\mu_{is} W_{iHs,t-1}^{1-\rho} + (1-\mu_{is}) W_{iLs,t-1}^{1-\rho}} \dot{W}_{iLs,t}^{1-\rho}.$$

The sectoral price index in market j and sector s is

$$\begin{aligned}\dot{P}_{js,t}^{1-\sigma_s} = & \sum_{i \neq j} \lambda_{ij,s,t-1} \left[\frac{\dot{\tau}_{ijs,t}}{\dot{T}_{is,t}} \dot{W}_{is,t}^{\alpha_{is}} \prod_k \dot{P}_{ik,t}^{\alpha_{iks}} \right]^{1-\sigma_s} \\ & + \lambda_{jj,s,t-1}^{hp} \left[\frac{\dot{\tau}_{jjs,t}}{\dot{T}_{js,t}} \dot{W}_{js,t}^{\alpha_{js}} \prod_k \dot{P}_{jk,t}^{\alpha_{jks}} \right]^{1-\sigma_s} + \lambda_{jj,s,t-1}^{lp} \left[\frac{(\dot{W}_{js,lp,t})^{\alpha_{js}}}{\dot{T}_{js,lp,t}} \prod_k \dot{P}_{jk,t}^{\alpha_{jks}} \right]^{1-\sigma_s}.\end{aligned}$$

Trade shares and goods market clearing.

For $i \neq j$, the Armington trade shares evolve as

$$\lambda_{ij,s,t} = \frac{\lambda_{ij,s,t-1} \left[\frac{\dot{\tau}_{ijs,t}}{\dot{T}_{is,t}} \dot{W}_{is,t}^{\alpha_{is}} \prod_k \dot{P}_{ik,t}^{\alpha_{iks}} \right]^{1-\sigma_s}}{\dot{P}_{js,t}^{1-\sigma_s}}.$$

For domestic sales ($i = j$), there are two types of firms. The Armington (high-productivity) domestic share is

$$\lambda_{ii,s,t}^{hp} = \frac{\lambda_{ii,s,t-1}^{hp} \left[\frac{\dot{\tau}_{iis,t}}{\dot{T}_{is,t}} \dot{W}_{is,t}^{\alpha_{is}} \prod_k \dot{P}_{ik,t}^{\alpha_{iks}} \right]^{1-\sigma_s}}{\dot{P}_{is,t}^{1-\sigma_s}},$$

and the local (low-productivity) domestic share is

$$\lambda_{ii,s,t}^{lp} = \frac{\lambda_{ii,s,t-1}^{lp} \left[\frac{\dot{\tau}_{iis,t}}{\dot{T}_{is,t}} \dot{W}_{is,lp,t}^{\alpha_{is}} \prod_k \dot{P}_{ik,t}^{\alpha_{iks}} \right]^{1-\sigma_s}}{\dot{P}_{is,t}^{1-\sigma_s}}.$$

Total domestic share is then

$$\lambda_{ii,s,t} = \lambda^{hp}_{ii,s,t} + \lambda^{lp}_{ii,s,t}.$$

Aggregate expenditure and revenues satisfy the same goods-market-clearing structure as in the baseline model:

$$P_j C_j = \sum_s \dot{W}_{jHs,t} \dot{L}_{jHs,t} W_{jHs,t-1} L_{jHs,t-1} + \sum_s \dot{W}_{jLs,t} \dot{L}_{jLs,t} W_{jLs,t-1} L_{jLs,t-1} + \Delta_{jt},$$

$$R_{is,t} = \sum_{j=1}^{N} \lambda_{ij,s,t} \left(\sum_k \alpha_{jsk} R_{jk,t} + \phi_{js} P_j C_j \right).$$

Once aggregate sectoral revenue $R_{is,t}$ is determined, I recover revenues by firm type as

$$R^{lp}_{is,t} = \lambda^{lp}_{ii,s,t} E_{is,t}, \qquad R^{hp}_{is,t} = R_{is,t} - R^{lp}_{is,t},$$

where $E_{is,t} = \sum_k \alpha_{isk} R_{ik,t} + \phi_{is} P_i C_i$ denotes total expenditure on sector-s varieties in region i.

Labor Market Clearing.

Using $R^{hp}_{is,t}$ and $R^{lp}_{is,t}$, labor demand by skill group is

$$\begin{aligned} \dot{W}_{iLs,t} \dot{L}_{iLs,t} W_{iLs,t-1} L_{iLs,t-1} = (1-\mu_{is}) \left(\frac{\dot{W}_{is,t} W_{is,t-1}}{\dot{W}_{iLs,t} W_{iLs,t-1}} \right)^{\rho-1} \alpha_{is} R^{hp}_{is,t} \\ + \left(1 - \kappa \mu_{is}\right) \left(\frac{\dot{W}_{is,lp,t} W_{is,lp,t-1}}{\dot{W}_{iLs,t} W_{iLs,t-1}} \right)^{\rho-1} \alpha_{is} R^{lp}_{is,t}, \end{aligned}$$

$$\begin{aligned} \dot{W}_{iHs,t} \dot{L}_{iHs,t} W_{iHs,t-1} L_{iHs,t-1} = \mu_{is} \left(\frac{\dot{W}_{is,t} W_{is,t-1}}{\dot{W}_{iHs,t} W_{iHs,t-1}} \right)^{\rho-1} \alpha_{is} R^{hp}_{is,t} \\ + \kappa \mu_{is} \left(\frac{\dot{W}_{is,lp,t} W_{is,lp,t-1}}{\dot{W}_{iHs,t} W_{iHs,t-1}} \right)^{\rho-1} \alpha_{is} R^{lp}_{is,t}. \end{aligned}$$

The rigidity system and the definition of real income in hats are unchanged:

$$L_{igs,t-1}\dot{L}_{igs,t} \leq N_{igs,t}, \qquad \dot{W}_{igs,t} \geq \delta_g \dot{e}_{i,US,t} \quad \text{for } g \in \{H, L\},$$

$$\dot{w}_{j,g,s,t} = \frac{\dot{W}_{j,g,s,t}}{\dot{P}_{jg,t}} \frac{\dot{L}_{j,g,s,t}}{\dot{N}_{j,g,s,t}}.$$

Migration choice.

The migration block in hat algebra is:

$$\mu^g_{j,s,k,t} = \frac{\mu^g_{j,s,k,t-1}\, \dot{u}^{\beta/\nu_g}_{j,g,k,t+1}}{\sum_{h=0}^{S} \mu^g_{j,s,h,t-1}\, \dot{u}^{\beta/\nu_g}_{j,g,h,t+1}},$$

$$\dot{u}_{j,g,s,t} = \dot{w}_{j,g,s,t} \left(\sum_{k=0}^{S} \mu^g_{j,s,k,t-1}\, \dot{u}^{\beta/\nu_g}_{j,g,k,t+1} \right)^{\nu_g},$$

$$N_{j,g,s,t} = \sum_{k=0}^{S} N_{j,g,k,t-1}\, \mu^g_{j,k,s,t-1}.$$

Treatment of period 2.

Because period 1 corresponds to a steady state with no anticipation of the shock, mobility choices in period 1 reflect steady-state continuation values. In period 2, however, workers observe the shock unexpectedly, so the usual hat-algebra relations do not apply between period 1 and period 2. As in CDP, the correct hat relation for utility in period 2 is

$$\dot{u}^{shock}_{j,s,2} = \dot{\omega}^{shock}_{j,s,2} \left[\sum_{k=0}^{S} \mu_{j,s,k,1} \left(\dot{u}^{shock}_{j,k,3} \right)^{\beta/\nu_g} \left(\dot{u}^{shock}_{j,k,2} \right)^{\beta/\nu_g} \right]^{\nu}.$$

Similarly, the period-2 migration shares satisfy

$$\mu_{j,s,k,2} = \frac{\mu_{j,s,k,1} \left(\dot{u}^{shock}_{j,k,2} \right)^{\beta/\nu_g} \left(\dot{u}^{shock}_{j,k,3} \right)^{\beta/\nu_g}}{\sum_{h=0}^{S} \mu_{j,s,h,1} \left(\dot{u}^{shock}_{j,h,2} \right)^{\beta/\nu_g} \left(\dot{u}^{shock}_{j,h,3} \right)^{\beta/\nu_g}}.$$

7 Data

This section summarizes the dataset used in the quantitative analysis, with full details provided in Appendix A. The final dataset combines information on trade flows, production, and employment for 50 U.S. states, 36 individual foreign countries, and an aggregated "rest of the world", yielding a total of 87 regions over the period 2000–2007, all sourced from the WIOD 2013 release.

The analysis is conducted at the level of 16 market sectors. Following Rodríguez-Clare et al. (2022) and Caliendo et al. (2019), I distinguish 12 manufacturing sectors, one agricultural sector, and three service sectors, which I classify as skilled services, unskilled services, and other services.

Bilateral trade flows. I construct a sector–region bilateral trade flow matrix covering 50 U.S. states and 37 foreign regions. The procedure consists of five steps.

First, I compute bilateral sectoral trade flows between foreign countries directly from WIOD 2013. This yields a complete 37×37 matrix of country-by-country flows for each sector.

Second, I construct bilateral manufacturing trade flows among U.S. states. To do so, I combine the regional trade shares from the replication package of Caliendo et al. (2019) with WIOD 2013. From WIOD, I obtain total U.S. domestic sales by sector, and I scale the state-level trade shares from Caliendo et al. (2019) so that aggregate domestic sales match the WIOD totals.

Third, I complete the manufacturing block by constructing trade flows between U.S. states and foreign countries. For this, I use the state–sector export and import flows from the U.S. Census Bureau's Merchandise Trade Statistics. State-level exports are available beginning in 2002, and imports beginning in 2008. Following RUV, I assume that each state's relative importance in exports (or imports) with each partner country is constant from the first year of availability back to 2000. I apply the same procedure to agriculture.

Fourth, I compute bilateral trade flows in services among U.S. states and between U.S. states and foreign regions. Following RUV, I combine state-level production and expenditure in services from the BEA Regional Economic Accounts, WIOD data, and bilateral distances to construct service-sector flows consistent with a gravity structure. The methodology is described in detail in Appendix A.

Finally, I construct bilateral agricultural trade flows among U.S. states. The procedure parallels the approach used for services, with the key difference that state-level expenditure in agriculture is not directly observed. For production, I combine the 2002 and 2007 Agriculture Censuses, the National Marine Fisheries Service Census, and data from the U.S. Energy Information Administration (EIA). Given production by state and sector and observed expenditure for all other sectors, I infer an implied state-level agricultural expenditure consistent with the model's accounting structure.

Labor and intermediate input shares. For each region i and sector s, I require measures of the labor share, intermediate-input shares, CES parameters, and consumption shares. I begin by computing intermediate-input shares for each country–sector pair directly from WIOD 2013. Under Cobb–Douglas specification, each intermediate-input coefficient corresponds to the ratio of intermediate inputs to total output in that sector. The national labor share is therefore given by one minus the sum of all intermediate-input shares.

I then construct labor shares at the U.S. state level. Using BEA data, I compute value added for each state by subtracting taxes and subsidies from state-level GDP. For each manufacturing sector, I then calculate the share of national value added produced in each state, and combine these shares with national sectoral value added from WIOD to obtain state–sector value added. Dividing this quantity by state–sector total production yields

$$\alpha_{is} = \frac{\text{Value Added}_{is}}{R_{is}}.$$

Following CDP, whenever the implied labor share exceeds one, I truncate it at one. Intermediate-input coefficients at the state level are then rescaled proportionally to ensure that the labor

share and intermediate shares sum to one, while preserving their relative magnitudes across U.S. states. For agriculture and services, I assume that each state's labor share equals the national labor share.

The next step is to split labor compensation between skilled and unskilled workers. Using the WIOD Socio-Economic Accounts for the year 2000, I compute skilled and unskilled labor compensation by country and sector, and choose μ_{is} to match these shares given parameters ρ and κ. The latter governs the relative skill intensity of low-productivity local firms compared with Armington firms. In section (4), in line with the empirical evidence, I show that exporters and larger firms employ a significantly higher share of skilled workers than non-exporters (Brambilla et al., 2012, 2017; Breinlich & Criscuolo, 2008; Oi & Idson, 1999; Criscuolo et al., 2021). Guided by this evidence, I choose κ such that high-productivity firms exhibit approximately a 5% higher proportion of skill workers than low-productivity firms. To recover μ_{is} for each U.S. state, I draw on Census 2000 microdata to allocate workers between skilled and unskilled categories. I compute the share of state–sector value added attributable to each skill group and multiply these shares by state–sector value added, yielding skilled and unskilled labor compensation at the sector–state level. For regions outside the US, I use WIOD and repeat the exact same process.

Finally, I allocate domestic revenues between low-productivity local firms and the representative Armington firm. For simplicity, I assume a common domestic revenue split across sectors and countries. Bernard et al. (2003) document that only 21% of plants export, exporters are 5.6 times larger than non-exporters, and exporters sell 4.8 times more to the U.S. market than non-exporters. Similarly, using manufacturing data for Mexico, Verhoogen (2008) shows that exporters hired 414.29 workers, while non-exporter firms hired 308.37 workers during the period from 1993 to 2001. Guided by this evidence, in the baseline exercise, I set the domestic market share of local firms to 40% and that of Armington firms to 60%.

Number of workers and initial wages. To construct the initial distribution of workers and wages, I begin by using WIOD to obtain the number of workers engaged in produc-

tion in each country by skill group (skilled vs. unskilled) for the year 2000. Given these employment counts and the corresponding labor compensation by skill group (constructed in the previous step), I recover initial equilibrium wages for all countries.

For U.S. states, I follow a parallel procedure. Using the 2000 IPUMS USA microdata, I compute total employment by state and sector and identify the shares of skilled and unskilled workers within each state–sector pair. I then apply these skill shares to total national employment in each sector from WIOD to allocate national employment across U.S. states in a manner consistent with both sources.

With state–sector employment levels for each skill group and the state–sector labor compensation constructed earlier, I recover the initial equilibrium wages for skilled and unskilled workers in each region using the initial equilibrium production structure.

Initial production. In a model with only countries, the available data and calibrated parameters map one-to-one into the equilibrium conditions. This is no longer the case when working with regional (state-level) data. Given the calibrated objects described above—such as bilateral trade shares and total expenditure by country—I recover region–sector production levels by choosing R_{is} to satisfy the gravity equation (4). The resulting production matrix aligns closely with observed data; Appendix A provides additional details on the procedure and on the empirical fit. [16]

Initial migration flows. To discipline sectoral mobility in the initial steady state, I use monthly CPS data on worker transitions across industries. Given the limited number of observations and to avoid sparse transition matrices with many zeros, I follow an approach similar to Rodríguez-Clare et al. (2022) and pool multiple years of data. I extend the window to 1995–2002 in order to retain a sufficiently large sample after splitting workers into skilled and unskilled groups.[17] Using these pooled data, I construct empirical transition probabilities between sectors and then adjust them to ensure that the implied initial allocation of workers corresponds to a steady-state equilibrium—i.e., there is no change in

[16] The correlation between the revenue data and the model generated revenue is higher than 0.999.

[17] The extended window is necessary because I estimate mobility separately for skilled and unskilled workers.

sectoral employment in the absence of shocks. The correlation between the adjusted transition matrix and the raw empirical mobility patterns exceeds 99%. For regions outside the United States, I assume no sectoral mobility frictions. Additional details on the construction of the transition matrix and the steady-state adjustment procedure are provided in Appendix C.

Not in the labor force. For U.S. regions, I compute the share of individuals not in the labor force directly from Census 2000 microdata, distinguishing skilled and unskilled groups using the same education-based classification applied elsewhere in the paper. For foreign countries, I rely on labor-force participation rates from the World Bank. To obtain skill-specific non-participation rates, I use the World Bank's educational attainment categories and map them into unskilled (basic education) and skilled (advanced education). The non-participation rate for each skill group is then constructed as one minus the corresponding labor-force participation rate. This provides a consistent measure of the mass of individuals who remain outside the labor force across all regions in the baseline equilibrium.

8 Calibration

This section describes the calibration of the model. I discipline three key components: (i) the evolution of China's productivity over time and across sectors, (ii) the degree of downward nominal wage rigidity by skill group, and (iii) the elasticities governing migration responses of skilled and unskilled workers. Unless otherwise stated, the calibration targets moments over the period 2000–2007.

China shock. China's productivity is modeled as the product of an aggregate, time-varying component and a sector-specific component,

$$T_{t,s} = T_t \, T_s.$$

These two components are calibrated separately using variation in U.S. imports from China. I first discipline the aggregate productivity path $\{T_t\}$ using the time-series evolution of U.S. imports from China. As in Autor et al. (2013), I construct a leave-out measure of the China shock using a set of high-income comparison countries[18] and estimate the regression

$$\Delta X_{C,US,t} = a + b_1 \Delta X_{C,OC,t} + \varepsilon_t,$$

where $\Delta X_{C,US,t}$ denotes the change in U.S. imports from China at time t, and $\Delta X_{C,OC,t}$ denotes the corresponding change for the comparison group. Let $\Delta \hat{X}_{C,US,t}$ denote the fitted values from this regression. I choose the aggregate productivity sequence $\{T_t\}$ so that the model reproduces this predicted time path of U.S. imports from China.

I next calibrate the sectoral productivity terms $\{T_s\}$ to match the cross-sectional variation in import growth across sectors. Specifically, I estimate

$$\Delta X_{C,US,s}^{2007-2000} = b_1 \Delta X_{C,OC,s}^{2007-2000} + \varepsilon_s,$$

where $\Delta X_{C,US,s}^{2007-2000}$ denotes the change in U.S. imports from China in sector s between 2000 and 2007, and $\Delta X_{C,OC,s}^{2007-2000}$ is the corresponding change for the comparison group. Let $\Delta \hat{X}_{C,US,s}^{2007-2000}$ denote the fitted values. I then choose $\{T_s\}$ so that the model-implied changes in U.S. imports from China by sector match these predicted values.

Overall, the calibration matches seven aggregate moments governing the time path of import growth and sixteen sectoral moments governing its cross-sectional distribution, using seven aggregate and sixteen sector-specific productivity parameters, respectively.

Trade elasticity. In the baseline calibration, I assume a common trade elasticity across sectors and set $\sigma_s = 6$ for all s, a value standard in the quantitative trade literature.

[18] Australia, Germany, Denmark, Spain, Finland, and Japan.

Wage rigidity and migration elasticities. I jointly calibrate the downward nominal wage rigidity parameters and the migration elasticities for skilled and unskilled workers to match key reduced-form labor market effects of the China shock estimated by Autor et al. (2013). Specifically, I target four moments: a \$1,000 increase in Chinese imports per worker leads to (i) an increase in unemployment of 0.119 percentage points for skilled workers and 0.282 percentage points for unskilled workers, and (ii) an increase in the share of individuals not in the labor force (NILF) of 0.304 percentage points for skilled workers and 0.831 percentage points for unskilled workers.

I choose the wage rigidity parameters δ_g and the migration elasticities by skill group so that the model's response to the calibrated China shock matches these four moments. In the baseline calibration, this procedure yields migration elasticities of $\nu_{low} \approx 0.07$ and $\nu_{high} \approx 0.48$, and wage rigidity parameters of $\delta_{low} \approx 0.994$ and $\delta_{high} \approx 0.98$. Finally, I set $\kappa = 0.90$, implying that high-productivity firms employ, on average, approximately 5% more skilled workers than low-productivity firms.

Given the calibrated parameters, the next section studies the welfare implications of the China shock through the lens of the model.

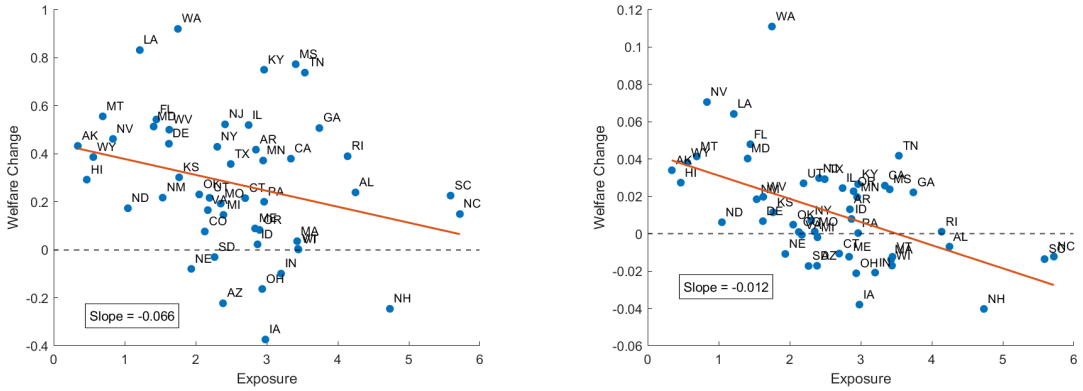
9 Welfare Effects

Following CDP, in Appendix D we show that the life-time welfare change for low-skilled and high-skilled workers in each (initial) sector and state can be computed using the following equation:

$$\ln(\Delta_{js}^{w}) = \sum_{t=1}^{\infty} \beta^{t} \left[\ln\left(\dot{\omega}_{jst}\right) - \nu \ln\left(\dot{\mu}_{jss,t}\right) \right].$$

Then, for my baseline calibration, Figure 7 shows the average welfare gains at the state level for each type of worker. A clear distinction is evident between the two types of workers. In particular, the welfare implications for high-skill workers are orders of magnitude

bigger in comparison with low-skill workers. On average, high-skill workers face an increase of 0.31%, while low-skill workers face an almost null welfare improvement, with an average of 0.01%.



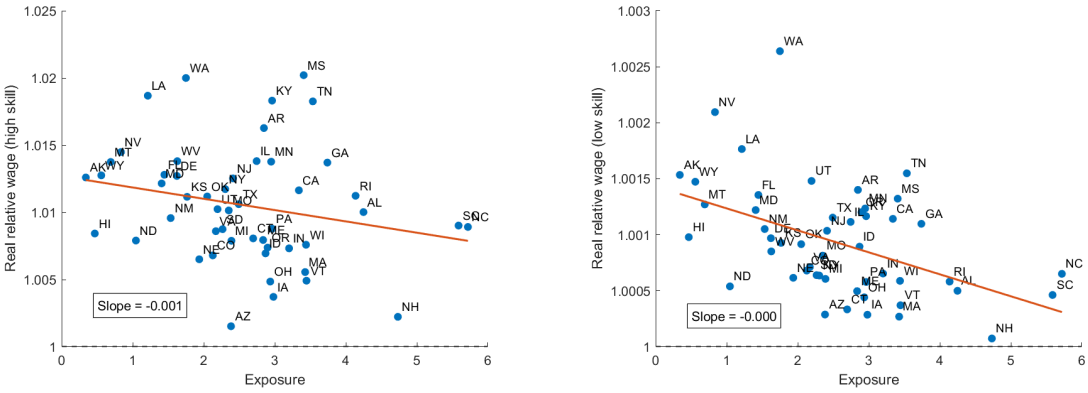
(a) Skilled-workers (b) Unskilled-workers

Figure 7: Welfare Change

Similarly, we can analyze the impact on long-run wages. Taking the initial distribution of workers across sectors in each state, Figure 8 shows the weighted-average long-run effect on wages for each state. We can see a clear difference between skill levels. I find an increase of real wages between 0.5% and 2% for most of the states with respect to high-skill workers. However, the real wage increment for low-skill workers is almost null. As expected, the negative effects on welfare come from the short-run rigidities. In particular, the unemployment-to-population rate of the United States evolves as shown in Figure 9. We can see a sizable difference between the unemployment-to-population rate of high skill workers and low skill workers in terms of magnitude and persistence.

Finally, Figure 10 shows the change in the risk premium between the initial year (2000) and the final year of the shock (2007). On average, the skill premium increases 0.78% in all states and, for mainland states, an increase of 1 in the exposure to the China Shock increases the skill premium by 0.54 $\%$.

Table 3 compares the empirical estimates with the moments generated by the model.



(a) Skilled-workers (b) Unskilled-workers

Figure 8: Long-Run Wage Change

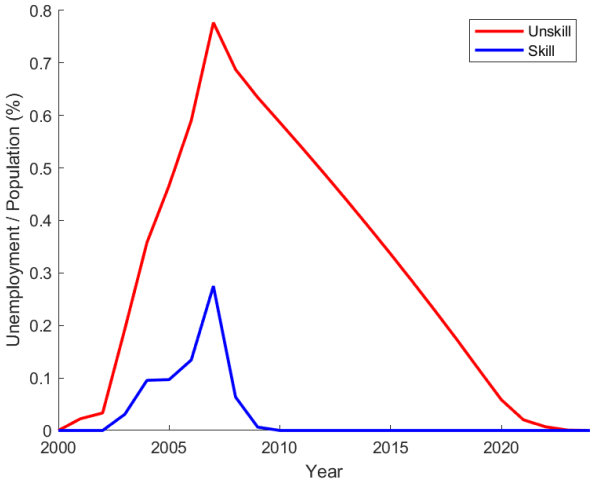


Figure 9: Unemployment Evolution

By construction, the model matches the targeted moments in terms of unemployment and not in the labor force participation change. It is worth mentioning that the non-targeted moments are not far from the point estimates. In particular, they lie inside the empirical 95% confidence intervals, with the exception of the manufacturing employment change for low skill workers, for which the moment generated by the model is slightly lower. I run several alternative specifications. The column *Lower* κ assumes that low-productivity firms have a share of skill workers that is 11% less than high-productivity firms, in contrast with our baseline estimation of 5%. However, the results do not change significantly. The mobility elasticity for low-skilled workers increases from 0.067 to 0.071, and most of the moments remain constant. The weighted-average skill premium increases from 0.78% to 0.79%. This implies that high-productivity firms face a similar contraction in their residual demands, despite having access to international markets.

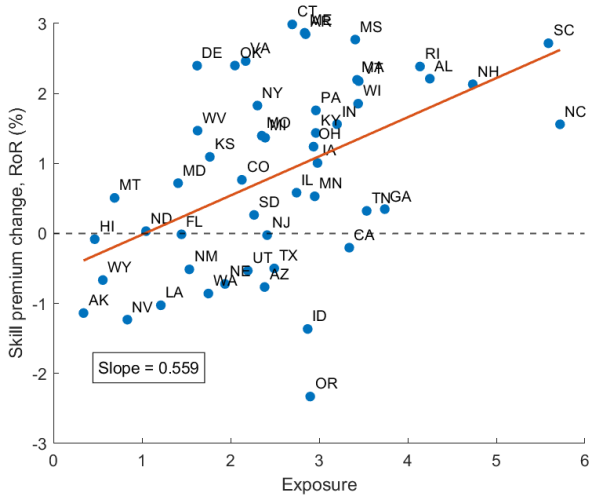


Figure 10: Skill-Premium

Table 3: Empirical Moments vs. Baseline Model

Moment	**Empirics**	**Empirics (CI)**	**Baseline Model**	**Lower κ**
Unemployment low-skill (t)	0.282*	[0.11,0.45]	0.282	0.282
Unemployment high-skill (t)	0.119*	[0.04,0.2]	0.119	0.119
NILF low-skill (t)	0.831*	[0.42,1.24]	0.831	0.831
NILF high-skill (t)	0.304*	[0.08,0.53]	0.304	0.304
Manufacturing emp. low-skill (nt)	-0.581*	[-0.77,-0.40]	-0.864	-0.864
Manufacturing emp. high-skill (nt)	-0.592*	[-0.84 ,-0.35]	-0.371	-0.368
Non-Mfg emp. low-skill (nt)	-0.531*	[-0.93,-0.13]	-0.249	-0.25
Non-Mfg emp. high-skill (nt)	0.168	[-0.07, 0.41]	-0.052	-0.055
Skill Premium (nt)	1.7%*	[0.8%, 2.6%]	0.55%	0.54%
Weighted avg. Welfare low-skill	-	-	0.01%	0.02%
Weighted avg. Welfare high-skill	-	-	0.31%	0.31%
ν_{low}	-	-	0.07	0.07
ν_{high}	-	-	0.47	0.47
δ_{low}	-	-	0.994	0.994
δ_{high}	-	-	0.98	0.98
κ	-	-	0.90	0.75
Low-Productivity share	-	-	0.4	0.4
Rigidity	-	-	Mfg	Mfg

*NOTE: we denote (t) for targeted moments and (nt) for non-targeted moments. The empirical moments come from Autor et al. (2013) and section (3). Variables with * are statistically significant, and the confidence interval reported is the 95% CI.*

10 Conclusion

This paper develops a dynamic quantitative trade model with wage rigidities, sectoral mobility frictions, and heterogeneity at both the firm and worker level to study the distributional consequences of trade shocks. Building on the dynamic structure in Rodríguez-Clare et al. (2022), I introduce a CES demand for labor at the firm level together with a positive relationship between firm productivity and the share of skilled workers employed within a sector. This extension allows the model to combine, in a tractable dynamic framework, heterogeneity on the firm side and on the worker side within each sector and region, bringing the model closer to the empirical patterns emphasized in the literature and to the kind of quantitative discipline typically pursued in dynamic trade models.

The key quantitative implication is that the China Shock has markedly different welfare consequences across skill groups. In line with the empirical evidence, the model predicts that unskilled workers experience essentially no welfare gains on average, and that their long-run real wage remains broadly unchanged. In contrast, skilled workers experience meaningful welfare gains, with an average welfare increase of about 0.30%, and long-run real wages rising between 1% and 3%. These gaps are consistent with the large differences in point estimates documented in Autor et al. (2013) and Autor et al. (2025), which highlights that the labor-market impacts of the China Shock differ substantially across worker types. In the model, these distributional differences arise despite the fact that both groups are exposed to the same aggregate trade disturbance: once firm heterogeneity and within-sector differences in skill usage are taken into account, the same trade-driven reallocation can translate into sharply different wage and welfare outcomes across workers.

Beyond the headline results, the analysis underscores a broader lesson for quantitative trade models used to assess winners and losers from globalization. Treating labor as homogeneous can obscure the fact that trade shocks interact with within-sector firm heterogeneity and skill composition in a way that changes the mapping from trade exposure to wage and welfare outcomes. By embedding worker heterogeneity and firm-level labor demand into a dynamic setting with wage rigidities and mobility frictions, the framework

provides a disciplined way to quantify these distributional effects over time, while remaining compatible with dynamic solution methods used in the literature.

The quantitative exercise also has limitations that point to promising directions for future work. First, the model uses a simplified firm structure with two firm types per sector. Second, workers within each sector-region are modeled as receiving an average of sector-region income, rather than explicitly allowing for a fraction of agents to be unemployed and earn zero income while the employed earn the market wage. Third, while the model includes frictions to sectoral reallocation, it abstracts from productivity costs of switching sectors and from agglomeration forces that could amplify or dampen the long-run effects of reallocation. These simplifications limit the range of mechanisms the model can speak to. At the same time, the framework remains useful for the purpose of this paper: it delivers a tractable quantitative laboratory that can isolate and quantify the starkly different welfare implications of trade shocks for skilled and unskilled workers. Overall, the results emphasize the importance of separating worker groups and evaluating distributional consequences in greater detail when using quantitative trade models to assess the effects of globalization.

References

Abraham, K. G. & Kearney, M. S. (2020). Explaining the decline in the us employment-to-population ratio: A review of the evidence. *Journal of Economic Literature*, 58(3), 585–643.

Accetturo, A., Bugamelli, M., & Lamorgese, A. R. (2013). Skill upgrading and exports. *Economics Letters*, 121(3), 417–420.

Acemoglu, D., Autor, D., Dorn, D., Hanson, G. H., & Price, B. (2016). Import competition and the great us employment sag of the 2000s. *Journal of Labor Economics*, 34(S1), S141–S198.

Adao, R., Arkolakis, C., & Esposito, F. (2019). *General equilibrium effects in space: Theory and measurement*. Technical report, National Bureau of Economic Research.

Alvarez, F. & Lucas Jr, R. E. (2007). General equilibrium analysis of the eaton–kortum model of international trade. *Journal of monetary Economics*, 54(6), 1726–1768.

Artuç, E., Chaudhuri, S., & McLaren, J. (2010). Trade shocks and labor adjustment: A structural empirical approach. *American economic review*, 100(3), 1008–1045.

Autor, D., Dorn, D., & Hanson, G. (2019). When work disappears: Manufacturing decline and the falling marriage market value of young men. *American Economic Review: Insights*, 1(2), 161–178.

Autor, D., Dorn, D., Hanson, G., Jones, M. R., & Setzler, B. (2025). Places versus people: the ins and outs of labor market adjustment to globalization. In *Handbook of Labor Economics*, volume 6 (pp. 549–653). Elsevier.

Autor, D., Dorn, D., & Hanson, G. H. (2021). *On the persistence of the China shock*. Technical report, National Bureau of Economic Research.

Autor, D., 'n, D., Hanson, G., & Majlesi, K. (2020). Importing political polarization? the electoral consequences of rising trade exposure. *American Economic Review*, 110(10), 3139–3183.

Autor, D. H. & Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American economic review*, 103(5), 1553–1597.

Autor, D. H., Dorn, D., & Hanson, G. H. (2013). The china syndrome: Local labor market effects of import competition in the united states. *American economic review*, 103(6), 2121–2168.

Autor, D. H., Dorn, D., & Hanson, G. H. (2016). The china shock: Learning from labor-market adjustment to large changes in trade. *Annual review of economics*, 8, 205–240.

Autor, D. H., Dorn, D., Hanson, G. H., & Song, J. (2014). Trade adjustment: Worker-level evidence. *The Quarterly Journal of Economics*, 129(4), 1799–1860.

Bernard, A. B., Eaton, J., Jensen, J. B., & Kortum, S. (2003). Plants and productivity in international trade. *American economic review*, 93(4), 1268–1290.

Bernard, A. B. & Jensen, J. B. (1997). Exporters, skill upgrading, and the wage gap. *Journal of international Economics*, 42(1-2), 3–31.

Boehm, C., Flaaen, A. B., Pandalai-Nayar, N., & Schlupp, J. (2025). *The Local-Area Incidence of Exporting*. Working Paper 34508, National Bureau of Economic Research.

Borusyak, K., Hull, P., & Jaravel, X. (2022). Quasi-experimental shift-share research designs. *The Review of economic studies*, 89(1), 181–213.

Brambilla, I., Lederman, D., & Porto, G. (2012). Exports, export destinations, and skills. *American Economic Review*, 102(7), 3406–3438.

Brambilla, I., Lederman, D., & Porto, G. (2017). Exporters, engineers, and blue-collar workers. *the world bank economic review*, 30(Supplement_1), S126–S136.

Breinlich, H. & Criscuolo, C. (2008). *Service Traders in the UK*. Number 901. London School of Economics and Political Science. Centre for Economic ....

Burstein, A., Morales, E., & Vogel, J. (2019). Changes in between-group inequality: computers, occupations, and international trade. *American Economic Journal: Macroeconomics*, 11(2), 348–400.

Burstein, A. & Vogel, J. (2017). International trade, technology, and the skill premium. *Journal of Political Economy*, 125(5), 1356–1412.

Bustos, P. (2011). Trade liberalization, exports, and technology upgrading: Evidence on the impact of mercosur on argentinian firms. *American economic review*, 101(1), 304–340.

Caliendo, L., Dvorkin, M., & Parro, F. (2019). Trade and labor market dynamics: General equilibrium analysis of the china trade shock. *Econometrica*, 87(3), 741–835.

Caliendo, L. & Parro, F. (2015). Estimates of the trade and welfare effects of nafta. *The Review of Economic Studies*, 82(1), 1–44.

Chetverikov, D., Larsen, B., & Palmer, C. (2016). Iv quantile regression for group-level treatments, with an application to the distributional effects of trade. *Econometrica*, 84(2), 809–833.

Costinot, A., Sarvimäki, M., & Vogel, J. (2022). *Evidence from the Collapse of Finnish-Soviet Trade*. Technical report, MIT working paper.

Criscuolo, C., Gal, P., Leidecker, T., & Nicoletti, G. (2021). The human side of productivity: Uncovering the role of skills and diversity for firm productivity.

Dekle, R., Eaton, J., & Kortum, S. (2007). Unbalanced trade. *American Economic Review*, 97(2), 351–355.

Dix-Carneiro, R. & Kovak, B. K. (2017). Trade liberalization and regional dynamics. *American Economic Review*, 107(10), 2908–2946.

Dix-Carneiro, R. & Kovak, B. K. (2019). Margins of labor market adjustment to trade. *Journal of International Economics*, 117, 125–142.

Dix-Carneiro, R., Pessoa, J. P., Reyes-Heroles, R., & Traiberman, S. (2023). Globalization, trade imbalances, and labor market adjustment. *The Quarterly Journal of Economics*, 138(2), 1109–1171.

Eaton, J. & Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5), 1741–1779.

Erumban, A. A., Gouma, R., de Vries, G., de Vries, K., & Timmer, M. (2012). Sources for national supply and use table input files. *World Input-Output Database (WIOD), 2012b.*, 3.

Fally, T. (2015). Structural gravity and fixed effects. *Journal of international economics*, 97(1), 76–85.

Feenstra, R. C. (2015). *Advanced international trade: theory and evidence*. Princeton university press.

Fitzgerald, D. & Haller, S. (2014). *Exporters and shocks: Dissecting the international elasticity puzzle*. Technical report, National Bureau of Economic Research.

Fitzgerald, D., Haller, S., & Yedid-Levi, Y. (2024). How exporters grow. *Review of Economic Studies*, 91(4), 2276–2306.

Galle, S., Rodríguez-Clare, A., & Yi, M. (2023). Slicing the pie: Quantifying the aggregate and distributional effects of trade. *The Review of Economic Studies*, 90(1), 331–375.

Hazell, J., Herreno, J., Nakamura, E., & Steinsson, J. (2022). The slope of the phillips curve: evidence from us states. *The Quarterly Journal of Economics*, 137(3), 1299–1344.

Head, K. & Mayer, T. (2014). Gravity equations: Workhorse, toolkit, and cookbook. In *Handbook of international economics*, volume 4 (pp. 131–195). Elsevier.

Head, K., Mayer, T., et al. (2002). *Illusory border effects: Distance mismeasurement inflates estimates of home bias in trade*, volume 1. Citeseer.

Kim, B., de la Barrera, M., & Fukui, M. (2023). *Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock*. Technical report, MIT working paper.

Kleinman, B., Liu, E., & Redding, S. J. (2023). Dynamic spatial general equilibrium. *Econometrica*, 91(2), 385–424.

Lewis, D., Melcangi, D., & Pilossoph, L. (2024). *Latent heterogeneity in the marginal propensity to consume*. Technical report, National Bureau of Economic Research.

Lyon, S. & Waugh, M. E. (2019). Quantifying the losses from international trade. *Unpublished manuscript*.

Mayer, T. & Zignago, S. (2011). Notes on cepii's distances measures: The geodist database.

Mayer, T., Zignago, S., et al. (2005). *Market access in global and regional trade*. CEPII Paris.

McCallum, J. (1995). National borders matter: Canada-us regional trade patterns. *The American economic review*, 85(3), 615–623.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *econometrica*, 71(6), 1695–1725.

Oi, W. Y. & Idson, T. L. (1999). Firm size and wages. *Handbook of labor economics*, 3, 2165–2214.

Papke, L. E. & Wooldridge, J. M. (2008). Panel data methods for fractional response variables with an application to test pass rates. *Journal of econometrics*, 145(1-2), 121–133.

Pierce, J. R. & Schott, P. K. (2016). The surprisingly swift decline of us manufacturing employment. *American economic review*, 106(7), 1632–1662.

Rodríguez-Clare, A., Ulate, M., & Vasquez, J. P. (2022). *Trade with nominal rigidities: Understanding the unemployment and welfare effects of the china shock*. CESifo, Center for Economic Studies & Ifo Institute.

Schmitt-Grohé, S. & Uribe, M. (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy*, 124(5), 1466–1514.

Serti, F., Tomasi, C., & Zanfei, A. (2008). *Exporters, Importers and Two-way traders: The links between internationalization, skills and wage*. Technical report, LEM Working Paper Series.

Steger, T. M. (2000). Economic growth with subsistence consumption. *Journal of Development economics*, 62(2), 343–361.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39 (pp. 195–214).: Elsevier.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., & De Vries, G. J. (2015). An illustrated user guide to the world input–output database: the case of global automotive production. *Review of International Economics*, 23(3), 575–605.

Topalova, P. (2007). Trade liberalization, poverty and inequality: Evidence from indian districts. In *Globalization and poverty* (pp. 291–336). University of Chicago Press.

Verhoogen, E. A. (2008). Trade, quality upgrading, and wage inequality in the mexican manufacturing sector. *The quarterly journal of economics*, 123(2), 489–530.

Waugh, M. E. (2023). *Heterogeneous Agent Trade*. Technical report, National Bureau of Economic Research.

Wen, C.-H. & Koppelman, F. S. (2001). The generalized nested logit model. *Transportation Research Part B: Methodological*, 35(7), 627–641.

Zingher, J. N. (2022). Trends: Diploma divide: Educational attainment and the realignment of the american electorate. *Political Research Quarterly*, 75(2), 263–277.

A Data Construction

List of Countries. I follow closely RUV and Caliendo et al. (2019) and use data for 50 U.S. states, 37 other countries including a constructed rest of the world. The list of countries is: Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, the Slovak Republic, Slovenia, S. Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world. I add Luxembourg, Malta and Latvia to the rest of the world.

Data on bilateral trade between countries. World Input-Output Database (WIOD). Release of 2013. We use data for 2000-2007. I map the sectors in the database to our 16 sectors in the following way: **1)** Agriculture (c1) and c(2) - NAICS 11, 211 - 213; **2)** Food Products, Beverage, and Tobacco Products (c3) - NAIC 311 and 312; **3)** Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4-c5) - NAIC 313-316; **4)** Wood Products, Paper, Printing, and Related Support Activities (c6-c7) - NAIC 321 - 323; **5)** Petroleum and Coal Products (c8) - NAICS 324; **6)** Chemical (c9) - NAICS 325; **7)** Plastics and Rubber Products (c10) - NAICS 326; **8)** Nonmetallic Mineral Products (c11) - NAICS 327; **9)** Primary Metal and Fabricated Metal Products (c12) - 331-332; **10)** Machinery (c13) - NAICS 333; **11)** Computer and Electronic Products, and Electrical Equipment and Appliances (c14) - NAICS 334-335; **12)** Transportation Equipment (c15) - NAICS 336; **13)** Furniture and Related Products, and Miscellaneous Manufacturing (c16) - NAICS 337-339; **14)** Construction (c18), Wholesale and Retail Trade (c19-c21), Transport Services (c23-c26) - NAICS 23,42-45,481-488; **15)** Information Services (c27), Finance and Insurance (c28), Real Estate (c29- c30); Education (c32); Health Care (c33) - NAICS 511-518,521-525,531-544, 61, 621-624; **16)** Accommodation and Food Services (c22), and Other Services (c34) - NAICS 493, 541,55,561,562,711-713,811-814,721-722. I called the first service sector *unskilled* and the second *skilled*.

Data on bilateral trade in manufacturing between U.S states. I combined Caliendo

et al. (2019) regional trade shares and WIOD 2013. From the WIOD database, we compute the total U.S. domestic sales for our 12 sectors. And I take from Caliendo et al. (2019) the shares such that total sales add up to the WIOD 2013 dataset. For the year 2005-2007, I followed RUV and compute the shares using the closest CFS publication (2007). I follow Caliendo et al. (2019) and use Table 12 of CFS 2007 to move from SCTG to NAIC codes. For those NAIC codes in which a corresponding SCTG has a value of S, I replace it with 0. Then, for example, NAIC code 311 (Food Manufacture) is composed of 40% of SCTG code 7 (Other prepared foodstuffs and fats and oils), 28% of SCTG code 5 (Meat, fish, seafood, and their preparations), 17% by SCTG code 6 (Grains, alcohol, and tobacco products), 8% of SCTG code 4 (Animal feed and products of animal origin, nec) and the other 7% are the other codes.

Data on bilateral trade in manufacturing and agriculture between U.S states and the rest of the countries. I followed RUV and used Import and Export Merchandise Trade Statistics, which is compiled by the U.S. Census Bureau. Data for exports at the state×sector level starts in 2002. Data for imports at the state×sector level starts in 2008. I scale state-level imports and exports data from the Import and Export Merchandise Trade Statistics to match the U.S. totals in WIOD.

Data on bilateral trade in low-skill services. I follow RUV and combine WIOD 2013 (from 1995 to 2011) with a gravity structure for services and agriculture separately to compute trade flows between US states. I start with the standard gravity equation:

$$X_{ij,s} = \left(\frac{p_{ij,s}}{P_{j,s}}\right)^{1-\sigma_s} E_{js}$$

Where the optimal price is satisfied:

$$p_{js} = \frac{\tau_{ijs}}{T_{i,s}} W_{iHs}^{\alpha_{iHs}} W_{iLs}^{\alpha_{iLs}} \prod_k P_{ik}^{\alpha_{iks}} = \tau_{ijs} c_{i,s}$$

At the same time we know that:

$$R_{i,s} = \sum_j X_{ij,s}$$

Combining this expressions we have $R_{i,s} = \sum_j \left(\frac{\tau_{ij,s} c_{i,s}}{\left(\sum_{i=1}^N p_{ijs}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}} \right)^{1-\sigma_s} E_{j,s}$ and so we can write this as:

$$R_{i,s} = c_{is}^{1-\sigma_s} \underbrace{\sum_j \left(\frac{\tau_{ij,s}}{\left(\sum_{i=1}^N p_{ijs}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}} \right)^{1-\sigma_s} E_{j,s}}_{\tilde{\Pi}_{is}}$$

As a consequence, we have the exact same system of equations as in RUV:

$$\tilde{\Pi}_{is} = \sum_j \tilde{\tau}_{ij,s} \tilde{P}_{js}^{-1} E_{js}$$

$$\tilde{P}_{js} = \sum_i \tilde{\tau}_{ij,s} \tilde{\Pi}_{is}^{-1} R_{is}$$

where every variable $\tilde{x}$ implies $x^{1-\sigma}$. The second expression came from the fact that $P_{js}^{1-\sigma_s} = \tilde{P}_{js} = \sum_{i=1}^N p_{ijs}^{1-\sigma_s} = \sum_{i=1}^N \tau_{ij}^{1-\sigma_s} c_{i,s}^{1-\sigma_s} = \sum_{i=1}^N \tilde{\tau}_{ij} R_{i,s} \tilde{\Pi}_{i,s}$. Again, given E_{is}, R_{is} and $\tilde{\tau}_{ij,s}$, this has a unique solution up to a constant (Fally (2015)). This indeterminacy requires a normalization.

As inputs, I need total expenditures in services for each region and total production in services. For the case of countries, I take this directly from WIOD. For the case of U.S. states, I take these variables from the Regional Economic Accounts of the Bureau of Economic Analysis. I scale the state-level services production and expenditures so that they aggregate to the U.S. totals in WIOD. While for the production of services, BEA generates a detailed description of them, this is not the case for expenditure. To solve this problem, I make a proportionality assumption: each state consumes the same share of each type of service as the US.

Given the solution to the system, we can calculate export-import flows:

$$X_{ij,s} = \tilde{P}_{j,s}^{-1} E_{j,s} \hat{p}_{ij,s}^{1-\sigma_s} = \tilde{P}_{j,s}^{-1} E_{j,s} R_{i,s} \tilde{\Pi}_{i,s}^{-1} \tilde{\tau}_{ij}$$

However, I need to calibrate the trade costs. I impose the following extended structure:

$$\tau_{ij,s}^{1-\sigma_s} = \tilde{\tau}_{ij,s} = \beta_0 \beta_1^{\iota_{ij}} distance_{ij}^{\beta_2} exp(\epsilon_{ij,s})$$

And so $log(\tilde{\tau}_{ij,s}) = log\ \beta_0 + \iota_{ij} log\ \beta_1 + \beta_2\ log(distance_{ij}) + \epsilon_{ij,s}$, where ι_{ij} in an indicator equal to 1 if i and j are in the same country and 0 otherwise. To calculate the distance between two countries, I followed the GeoDist dataset of CEPI provided by Mayer & Zignago (2011). In particular, they used the following definition of distance between two countries:

$$dist_{ij} = \left(\sum_{r \in i} \sum_{s \in j} \left(\frac{pop_r}{pop_i} \right) \left(\frac{pop_s}{pop_j} \right) d_{rs}^{-1} \right)^{-1}$$

which corresponds to the formula that is traditionally used in the literature. I combined this formula with the assumption about the trade cost functional form and run a standard gravity equation regression. Table 4 has the results for several specifications.

For distances between countries (including the US), I used data from *simplemaps*[19]. Following the same methodology as Mayer & Zignago (2011), I take the 25 more populated cities for each country. The only two exceptions are Cyprus and Estonia, given that I have 14 and 20 cities in the dataset, respectively. For RoW, I take the 25 most populated cities that are not part of any country listed before and use this subset to run the regression mentioned before. In addition, I calculate distances between states in the US. I used the US Census population data in 2010 and take every county in the database. I used the Haversine distance to calculate the distance between regions. I aggregate distances between counties to states using the formula mentioned above. Finally, I calculate distances between US states and other countries. To do that, I combined *simplemaps* and the US Census Population data as mentioned before.

Given my estimates, I followed RUV and set the bilateral resistance term in each sector using the estimates from Model 4. For the US states, the only binding dummy is Domestic by construction. To solve the model, I need to calculate $R_{i,s}$ and $E_{i,s}$ for every sector s and every region/country i. For non-US countries, I used WIOD. For US state expenditures, I

[19] https://simplemaps.com/data/world-cities

Table 4: Estimation of Own-Country Dummy and Distance Elasticity for low skill service

Dependent Variable:	ln_Exports			
Model:	(1)	(2)	(3)	(4)
Variables				
log(distance)	-0.4132***	-0.5123***	-0.5892***	-0.5902***
	(0.0468)	(0.0396)	(0.0387)	(0.0399)
Domestic	7.458***	7.113***	6.825***	6.821***
	(0.1883)	(0.1680)	(0.1642)	(0.1681)
Common Language Dummy	-0.7216***	-0.6780***	0.0195	
	(0.1687)	(0.1681)	(0.1521)	
Share Border Dummy	0.7528***			
	(0.0786)			
Colonial Link Ever	1.686***	1.884***		
	(0.0940)	(0.0989)		
Fixed-effects				
origin_country_num-year_f	Yes	Yes	Yes	Yes
destination_country_num-year_f	Yes	Yes	Yes	Yes
Fit statistics				
Observations	24,548	24,548	24,548	24,548
Adjusted R^2	0.57640	0.57525	0.56914	0.56916

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

use the Regional Economic Accounts of the Bureau of Economic Analysis. Given that the decomposition across services is limited, I aggregate at the service level and assume that each state consumes shares of each type of service equal to the national proportions. For US state production, I also use the Bureau of Economic Analysis. I scale up or down the production and expenditure such that it matches the WIOD US data. Given that the data is disaggregated, I matched industry classification with the service sectors following the same procedure as before. Finally, we can rewrite the system of equations in a similar way to RUV:

$$\tilde{\Pi}_{is} = \sum_{j} \tilde{\tau}_{ij,s} \tilde{P}_{js}^{-1} E_{js} \quad i \in US$$

$$\tilde{P}_{js} = \sum_{i} \tilde{\tau}_{ij,s} \tilde{\Pi}_{is}^{-1} R_{is} \quad j \in US$$

$$\tilde{\lambda}_{is}^{*} \tilde{\Pi}_{is} = \sum_{j \in US} \tilde{\tau}_{ij,s} \tilde{P}_{js}^{-1} E_{js} \quad i \notin US$$

$$\tilde{\lambda}_{js} \tilde{P}_{js} = \sum_{i \in US} \tilde{\tau}_{ij,s} \tilde{\Pi}_{is}^{-1} R_{is} \quad j \notin US$$

Where $\tilde{\lambda}_{js}$ is the share of imports from the US and $\tilde{\lambda}_{is}^{*}$ is the share of exports to the US:

$$\tilde{\lambda}_{js} = \frac{\sum_{i \in US} X_{ij,s}}{E_{js}} \quad and \quad \tilde{\lambda}_{is}^{*} = \frac{\sum_{i \in US} X_{ij,s}}{R_{is}}$$

Given this new system of equations, we only need to calibrate $\tilde{\tau}_{ij,s}$ for the cases where i or j are in the US. Following the results of McCallum (1995), I assume that the value of the domestic dummy for States is equal to 1 if both the origin and the destination are the same region.

Once I find the solution for the same system of equations described above, I compute the final bilateral trade matrix for the sector.

Data on bilateral trade in high-skilled services between US states. I repeat the same procedure as before. The estimation of the gravity equations gives the following results:

Table 5: Estimation of Own-Country Dummy and Distance Elasticity for high skill service

Dependent Variable:	ln_Exports			
Model:	(1)	(2)	(3)	(4)
Variables				
log(distance)	-0.0076	-0.1099***	-0.1720***	-0.1904***
	(0.0470)	(0.0403)	(0.0392)	(0.0398)
Domestic	9.015***	8.659***	8.426***	8.354***
	(0.1927)	(0.1741)	(0.1700)	(0.1718)
Common Language Dummy	-0.2546*	-0.2096	0.3535***	
	(0.1471)	(0.1468)	(0.1306)	
Share Border Dummy	0.7770***			
	(0.0854)			
Colonial Link Ever	1.316***	1.521***		
	(0.0882)	(0.0910)		
Fixed-effects				
origin_country_num-year_f	Yes	Yes	Yes	Yes
destination_country_num-year_f	Yes	Yes	Yes	Yes
Fit statistics				
Observations	24,548	24,548	24,548	24,548
Adjusted R^2	0.60970	0.60845	0.60441	0.60419

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Once I find solutions for the same system of equations described above, I compute the final bilateral trade matrix for the sector.

Data on bilateral trade in other services for US states. Again, I follow the same

procedure as before. The gravity equation estimation is:

Table 6: Estimation of Own-Country Dummy and Distance Elasticity for other service

Dependent Variable:	ln_Exports			
Model:	(1)	(2)	(3)	(4)
Variables				
log(distance)	-0.1571***	-0.2176***	-0.2769***	-0.2980***
	(0.0571)	(0.0488)	(0.0482)	(0.0482)
Domestic	9.730***	9.519***	9.297***	9.214***
	(0.2370)	(0.2154)	(0.2126)	(0.2129)
Common Language Dummy	-0.1577	-0.1311	0.4065**	
	(0.1848)	(0.1833)	(0.1658)	
Share Border Dummy	0.4596***			
	(0.1312)			
Colonial Link Ever	1.331***	1.452***		
	(0.1429)	(0.1457)		
Fixed-effects				
origin_country_num-year_f	Yes	Yes	Yes	Yes
destination_country_num-year_f	Yes	Yes	Yes	Yes
Fit statistics				
Observations	24,548	24,548	24,548	24,548
Adjusted R^2	0.52361	0.52336	0.52112	0.52095

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Data on bilateral trade in agriculture for US states. I start by estimation the same regression as before:

Table 7: Estimation of Own-Country Dummy and Distance Elasticity for agriculture

Dependent Variable:	ln_Exports			
Model:	(1)	(2)	(3)	(4)
Variables				
log(distance)	-1.707***	-1.712***	-1.736***	-1.758***
	(0.0191)	(0.0163)	(0.0163)	(0.0165)
Domestic	2.245***	2.226***	2.137***	2.052***
	(0.1149)	(0.1097)	(0.1097)	(0.1111)
Common Language Dummy	0.2029***	0.2053***	0.4200***	
	(0.0730)	(0.0720)	(0.0778)	
Share Border Dummy	0.0411			
	(0.0522)			
Colonial Link Ever	0.5689***	0.5798***		
	(0.0655)	(0.0600)		
Fixed-effects				
origin_country_num-year_f	Yes	Yes	Yes	Yes
destination_country_num-year_f	Yes	Yes	Yes	Yes
Fit statistics				
Observations	24,548	24,548	24,548	24,548
Adjusted R^2	0.76984	0.76985	0.76909	0.76867

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

The main difference between services and agriculture is that I do not have data on state expenditure. For production, I combine the Agriculture Census from 2002 and 2007 with the National Marine Fisheries Service Census and the US Energy Information Administration (EIA). The National Marine Fisheries Service Census has annual data from 2000

to 2007. However, given that the Agriculture Census has data only for 2002 and 2007, I continue as follows: I compute total production for the year 2002 (2007) in each state for agriculture and fisheries. Then, I compute the share of production over the total. Given the shares, I multiply them by WIOD US production for the year 2002 (2007), only for the c1 category (which does not include mining). For those states that are missing in the National Marine Fisheries Service Census, I imputed a value of 0. Then, I need to compute the other component of the sector: mining. To do that, we proxy the share of sales for each state using the US Energy Information Administration (EIA). They provide data for the annual thousand of barrels for each state. Given that I only need the share, under the assumption that prices does not vary between states, we can compute the share of production for each state and multiply it by the national production level in the c2 category for US in WIOD. Finally, we add c1 and c2 categories to get total production of sector 1 for each state. I continue with the expenditure side. Total expenditure in region j - sector k is:

$$E_{jk} = \sum_s \alpha_{jks} R_{js} + \phi_{jk} \left(W_j L_j + \Delta_j \right)$$

For $j \in US$, we have:

$$E_{jk} = \sum_s \alpha_{jks} R_{js} + \phi_{USk} \left(W_j L_j + \Delta_j \right)$$

Note that we need to find an expression for $(W_j L_j + \Delta_j)$ and then we can solve the level of agricultural expenditure in each state j. To do that, note that household consumption is equal to $E_{jk} + \sum_{r \neq k} \left(E_{jr} - \sum_s \alpha_{jrs} R_{js} \right) - \sum_s \alpha_{jks} R_{js}$. As a result, we can write regional expenditure in agriculture (k = AG) as:

$$E_{j,AG} = \sum_s \alpha_{j,AG,s} R_{js} + \frac{\phi_{US,AG}}{1 - \phi_{US,AG}} \sum_{r \neq AG} \left(E_{jr} - \sum_s \alpha_{US,rs} R_{js} \right)$$

Given that we have production in each sector and state and we have expenditure for all the other sectors, we can compute the expenditure by state in agriculture. After calculating expenditure for each state, we can use a gravity system similar to that of services. However,

for agriculture we have data on imports and exports between US states and other countries. Then, we only need to calculate bilateral flows among US states. The system of equation simplifies to:

$$\tilde{\mu}_{is}^{*}\tilde{\Pi}_{is} = \sum_{j \in US} \tilde{\tau}_{ij,s}\tilde{P}_{js}^{-1}E_{js} \quad i \in US$$

$$\tilde{\mu}_{js}\tilde{P}_{js} = \sum_{i \in US} \tilde{\tau}_{ij,s}\tilde{\Pi}_{is}^{-1}R_{is} \quad j \in US$$

where:

$$\tilde{\mu}_{js} = \frac{\sum_{i \in US} X_{ij,s}}{E_{js}} \quad and \quad \tilde{\mu}_{is}^{*} = \frac{\sum_{j \in US} X_{ij,s}}{R_{is}}$$

Labor and intermediate input shares. For each region i and sector s, I need data to compute the labor shares, intermediate input shares, and consumption shares. I start by computing the intermediate input shares for each sector in each country from WIOD. Given that in our model there are no profits, the assumption of the Cobb-Douglas production function implies that total labor share ($\alpha_{is}^{WIOD} = \alpha_{jHs} + \alpha_{jLs}$) is equal to one minus the sum of all the intermediate input shares. For simplicity, I assume that US national intermediate input shares are common across US regions and so total labor shares are also equal. However, I allow different intensities between skilled and unskilled workers for each US region in all sectors. Then, I use WIOD to calculate the labor input share for skilled and unskilled workers separately. I compute $\alpha_{iLs}/\alpha_{iHs}$ as the share of total labor compensation for unskilled level against skilled labor. Then, I combine this equation with $\alpha_{iLs} + \alpha_{iHs} = \alpha_{is}^{WIOD}$ to pin down both coefficients for each country and sector. I include medium-skilled workers as unskilled workers to fit our framework. Note that WIOD does not provide data for RoW. To solve this problem, given that this is not a relevant region, I take the average for all the other countries in our exercise. For US states, I combine IPUMS data with my WIOD database.

Initial Production. Conditional on all the calibrated parameters mentioned before, the trade shares and total expenditure by country, I calculate sector-region production such that the model clears. In particular, I choose $R_{is,2000}$ such that the following equations are satisfied:

$$P_i C_i = W_{i,H} L_{i,H} + W_{i,L} L_{i,L} + \Delta_i$$

$$R_{i,s} = \sum_{j \notin US} \lambda_{ij,s} \phi_{js} C_j P_j + \phi_{US,s} \sum_{j \in US} \lambda_{ij,s} C_j P_j + \sum_{j=1}^{N} \lambda_{ij,s} \sum_{k=1}^{K} \alpha_{jsk} R_{jk}$$

$$W_{ig} L_{ig} = \sum_{s} \alpha_{i,g,s} R_{i,s}$$

$$\Delta_i = \sum_{n} X_{ni} - \sum_{n} X_{in} = E_i - \sum_{i} R_{is}$$

I assume that all shares are exogenous (calibrated), and I solve for sectoral production in each region such that this system of equations holds.

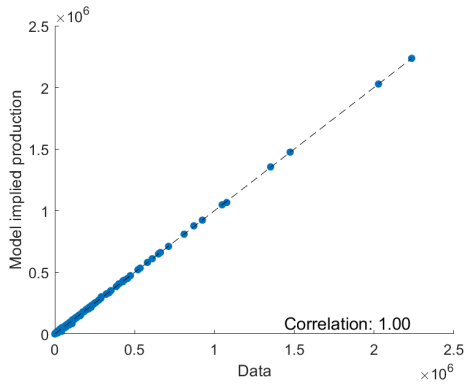


Figure 11: Production: Data vs Model Equilibrium

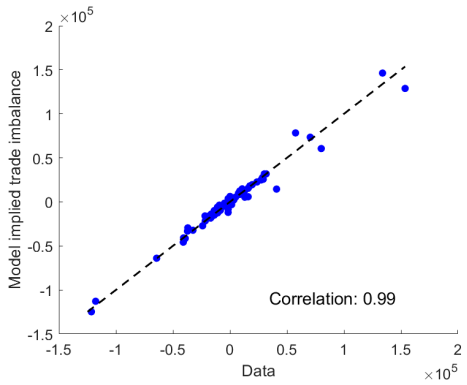


Figure 12: Unbalanced Trade: Data vs Model Equilibirum

B Appendix B: Exposure

In the data section we use the following expression to calculate the exposure of each type of worker:

$$\text{Exposure}_{ij} = \sum_{s=1}^{S} \frac{L_{ij,s,2000}}{L_{ij,2000}} \frac{\hat{X}^{2007-2000}_{C,US,s}}{R_{US,s,2000}}$$

Where $L_{ijs,2000}$ is the total employment of workers with skill level j in region i for sector s in the year 2000. In addition, $\hat{X}^{2007-2000}_{c,US,s}$ is the predicted difference in imports from China to US in sector s. To calculate this predicted change in imports, I follow Autor et al. (2013) and regress:

$$\Delta X^{2007-2000}_{C,US,s} = b_1 \Delta X^{2007-2000}_{C,\text{OC},s} + \epsilon_s$$

Following Autor et al. (2013) and RUV, we include Australia, Germany, Denmark, Spain, Finland, and Japan in the RHS of the regression. we define $\hat{X}^{2007-2000}_{c,US,s}$ as the predicted value of this regression. We can see the comparison between the predicted value and the observed change in US imports from China:

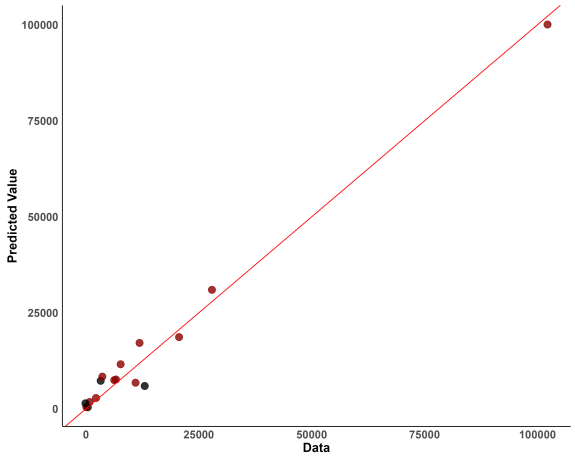


Figure 13: Predicted and observed change

The denominator is total sales in sectors. Then, I need to calculate employment for each skill level, region, and sector in the US. For this, I use two different datasets for robustness.

IPUMS CPS.I consider only people older than 24. I also exclude those who are out of the labor force (EMPSTAT higher than 29), without information (EMPSTAT 0), or in the army (EMPSTAT 13-15). I use EDUC99 to divide between high-skilled workers and low-skilled workers. In particular, If EDUC99 $<$ 12, the worker is unskilled, and if EDUC99 $\geq$ 12 the worker is skilled. This means that I consider high skilled worker

an adult with an associate degree or higher. I move to the occupation. I delete all the observations where IND is equal to 0 (occupation not specified) and any job related to the government (IND equal or higher than 900). Given that we do not have the electricity sector, I delete IND 450-451-452-470 and 472. I also delete the postal service (IND 412). Then, we need to match IND codes with our WIOD sectors; **Agriculture**: 10, 11, 30, 31, 32, 40, 41, 42, 50, 230. **Food Products, Beverage, and Tobacco Products**: 100,101,102,110,111,112,120,121,122,130. **Textile, Textile Product Mills, Apparel, Leather, and Allied Products**: 132,140,141,142,150,151,152. **Wood Products, Paper, Printing, and Related Support Activities**: 160,161,162,171,172,231,232,241. **Petroleum and Coal Products**: 200,201. **Chemical**: 180,181,182,190,191,192. **Plastics and Rubber Product**: 210,211,212,220,221,222. **Nonmetallic Mineral Products**: 250,251,252,261,262. **Primary Metal and Fabricated Metal Products**: 270, 271, 272, 280, 281, 282, 290, 291, 292, 300, 301. **Machinery**: 310,311,312,320,321,331,332,380. **Computer and Electronic Products, and Electrical Equipment and Appliances**: 322, 340, 341, 342, 350, 371, 381. **Transportation Equipment**: 351,352,360,361,362,370,500. **Furniture and Related Products, and Miscellaneous Manufacturing:** 242, 372, 390, 391, 392. **Unskilled Service**: 60, 400,401, 402,410, 420, 421, 422, 432, 501, 502,510, 511, 512, 521, 522, 530, 531, 532, 540, 541, 542 and any value between 542 and 691. **Skilled Service**: 440, 441, 442, 700, 701, 702, 710, 711, 712, 742, 800, 801, 820, 812, 821, 822, 830, 831, 832, 840, 842, 850, 851, 852, 860, 861, 862, 863, 870, 871. **Other Service**: 12, 20, 411, 471, 721, 722, 730, 731, 732, 740, 741, 750, 751, 752, 760, 761, 762, 770, 771, 772, 780, 781, 782, 790, 791, 802, 810, 841, 872, 873, 880, 881, 882, 890, 891, 892, 893. We have the following results for the exposure measure:

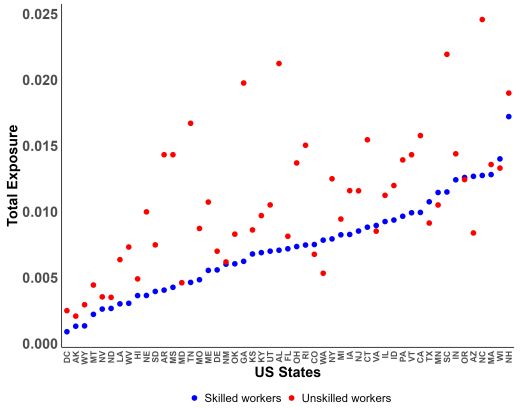


Figure 14: Exposure by state-skill level

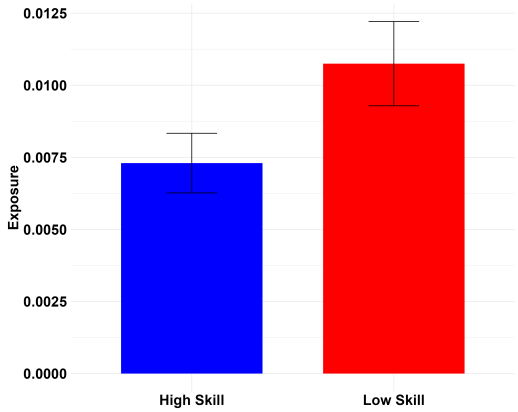


Figure 15: Exposure by skill level

IPUMS USA. Here we do not have the EDUC99 variable but we have INDNAIC, i.e. the occupation for each NAIC code. Then, we combine INDNAIC, EDUCD, and state data to replicate the same graph as before. I define a worker to be skilled (unskilled) if EDUCD is $\geq$ ($<$) than 80, i.e. if they have at least two years of college. Then, this is exactly the same as the filter I applied for EDUC99 in the sample mentioned before. I match all NAIC codes with my 16 sectors. The only exception is 3MS which corresponds to *Not specified industries*. We use this data to generate $L_{ijs,2000}/L_{ij,2000}$. Then we can create the same exposure measure as before:

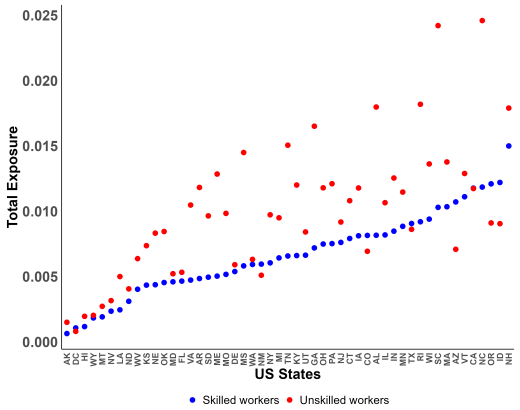


Figure 16: Exposure by state-skill level

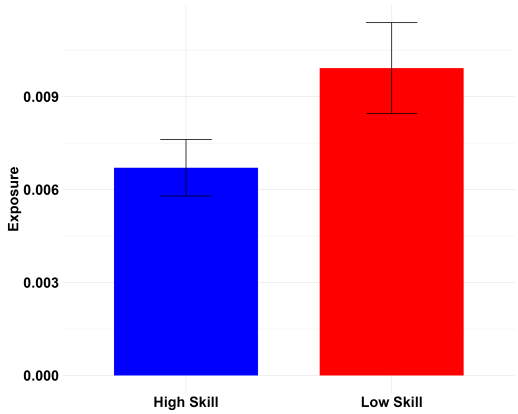


Figure 17: Exposure by skill level

C Appendix C: Migration choice

As mentioned in the paper, we denote period 1 as the steady state, and period 2 to 8 the shock period. Consider any period $t > 2$ and ignore skill level g. And denote $V_{jk,t}^{Shock}$ the present discount value after the shock is realized for any period t greater than 2.

$$\mu_{j,sk,t} = \frac{\exp\left(\beta V_{j,k,t+1}^{shock} - \kappa_{j,sk,g}\right)^{1/\nu_g}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,g,h,t+1}^{shock} - \kappa_{j,sh,g}\right)^{1/\nu_g}}$$

Taking period t and t-1 we have:

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(\beta V_{j,k,t+1}^{shock} - \kappa_{j,sk}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1}^{shock} - \kappa_{j,sh}\right)^{1/\nu}} \frac{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}}{\exp\left(\beta V_{j,k,t}^{shock} - \kappa_{j,sk}\right)^{1/\nu}}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(\beta V_{j,k,t+1}^{shock} - \beta V_{j,h,t}^{shock}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1}^{shock} - \kappa_{j,sh}\right)^{1/\nu}} \sum_{h=0}^{S} \exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(\beta V_{j,k,t+1}^{shock} - \beta V_{j,k,t}^{shock}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1}^{shock} - \kappa_{j,sh}\right)^{1/\nu}} \frac{\exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}}{\exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}} \sum_{h=0}^{S} \exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(\beta V_{j,k,t+1}^{shock} - \beta V_{j,k,t}^{shock}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1}^{shock} - \beta V_{j,h,t}^{shock}\right)^{1/\nu} \exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}} \sum_{h=0}^{S} \exp\left(\beta V_{j,h,t}^{shock} - \kappa_{j,sh}\right)^{1/\nu}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(\beta V^{shock}_{j,k,t+1} - \beta V^{shock}_{j,k,t}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V^{shock}_{j,h,t+1} - \beta V^{shock}_{j,h,t}\right)^{1/\nu} \frac{\exp\left(\beta V^{shock}_{j,h,t} - \kappa_{j,sh}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V^{shock}_{j,h,t} - \kappa_{j,sh}\right)^{1/\nu}}}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(V^{shock}_{j,k,t+1} - V^{shock}_{j,k,t}\right)^{\beta/\nu}}{\sum_{h=0}^{S} \exp\left(V^{shock}_{j,h,t+1} - V^{shock}_{j,h,t}\right)^{\beta/\nu} \frac{\exp\left(\beta V^{shock}_{j,h,t} - \kappa_{j,sh}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V^{shock}_{j,h,t} - \kappa_{j,sh}\right)^{1/\nu}}}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\exp\left(V^{shock}_{j,k,t+1} - V^{shock}_{j,k,t}\right)^{\beta/\nu}}{\sum_{h=0}^{S} \exp\left(V^{shock}_{j,h,t+1} - V^{shock}_{j,h,t}\right)^{\beta/\nu} \mu_{j,sh,t-1}}$$

$$\frac{\mu_{j,sk,t}}{\mu_{j,sk,t-1}} = \frac{\dot{u}^{\beta/\nu}_{jk,t+1}}{\sum_{h=0}^{S} \dot{u}^{\beta/\nu}_{jh,t+1} \mu_{j,sh,t-1}}$$

Now we have to be careful about period 2. In period 1 the agent does not predict the shock in period 2. This means that:

$$\mu_{j,sk,1} = \frac{\exp\left(\beta V^{no-shock}_{j,k,1} - \kappa_{j,sk}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V^{no-shock}_{j,h,1} - \kappa_{j,sh}\right)^{1/\nu}}$$

But in period 2 (first period of the shock):

$$\mu_{j,sk,2} = \frac{\exp\left(\beta V^{shock}_{j,k,3} - \kappa_{j,sk}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V^{shock}_{j,h,3} - \kappa_{j,sh}\right)^{1/\nu}}$$

Note that the life time utility $u_{jst} = exp(V_{jst})$ is equal to:

$$u_{jst} = \frac{\omega_{jst}}{P_{jt}} \left(\sum_{k=0}^{S} u_{jk,t+1}^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}$$

In period 1 there was no shock, so we have:

$$u_{js1}^{ns} = \frac{\omega_{js1}}{P_{j1}} \left(\sum_{k=0}^{S} (u_{jk,2}^{ns})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}$$

Denote $u_{js,2}^{shock}$ the life time utility in the period of the first shock when it's realized. Then we have:

$$u_{js1}^{ns} = \frac{\omega_{js1}^{ns}}{P_{j1}^{ns}} \left(\sum_{k=0}^{S} \left(\frac{u_{jk,2}^{ns}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} (u_{jk,2}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}$$

We can define the life time utility after the first shock as:

$$u_{js2}^{shock} = \frac{\omega_{js2}^{shock}}{P_{j2}^{shock}} \left(\sum_{k=0}^{S} (u_{jk,3}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}$$

Take the difference between the shock in 2 and the no shock in 1:

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{ns}/P_{j1}^{ns})} \frac{\left(\sum_{k=0}^{S} (u_{jk,3}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}}{\left(\sum_{k=0}^{S} \left(\frac{u_{jk,2}^{ns}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} (u_{jk,2}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu} \right)^{\nu}}$$

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{ns}/P_{j1}^{ns})} \left(\frac{\sum_{k=0}^{S} (u_{jk,3}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu}}{\sum_{k=0}^{S} \left(\frac{u_{jk,2}^{ns}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} (u_{jk,2}^{shock})^{\beta/\nu} (e^{\kappa_{jsk}})^{-1/\nu}} \right)^{\nu}$$

Note that we can rewrithe the migration flow in period 1 as:

$$\mu_{j,sk,1} = \frac{(u^{ns}_{jk,2})^{\beta/\nu}\ exp(\kappa_{j,sk})^{-1/\nu}}{\sum_{h=0}^{S}(u^{ns}_{jh,2})^{\beta/\nu}\ exp(\kappa_{j,sh})^{-1/\nu}} = \frac{\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}\ exp(\kappa_{j,sk})^{-1/\nu}}{\sum_{h=0}^{S}\left(\frac{u^{ns}_{jh,2}}{u^{shock}_{jh,2}}\right)^{\beta/\nu}(u^{shock}_{jh,2})^{\beta/\nu}\ exp(\kappa_{j,sh})^{-1/\nu}}$$

Going back to the lifetime ratio:

$$\frac{u^{shock}_{js2}}{u^{ns}_{js1}} = \frac{(\omega^{shock}_{js2}/P^{shock}_{j2})}{(\omega^{ns}_{js1}/P^{ns}_{j1})}\left(\frac{\sum_{k=0}^{S}(u^{shock}_{jk,3})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_{k=0}^{S}\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}\right)^{\nu}$$

$$\frac{u^{shock}_{js2}}{u^{ns}_{js1}} = \frac{(\omega^{shock}_{js2}/P^{shock}_{j2})}{(\omega^{ns}_{js1}/P^{ns}_{j1})}\left[\sum_{k=0}^{S}\left(\frac{(u^{shock}_{js,3})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_{k=0}^{S}\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{js,2}}\right)^{\beta/\nu}(u^{shock}_{js,2})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}\right)\right]^{\nu}$$

Multiply and divide each term in the sumation by $\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}$:

$$\frac{u^{shock}_{js2}}{u^{ns}_{js1}} = \frac{(\omega^{shock}_{js2}/P^{shock}_{j2})}{(\omega^{ns}_{js1}/P^{ns}_{j1})}$$
$$\left[\sum_{k=0}^{S}\left(\frac{(u^{shock}_{jk,3})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_{k=0}^{S}\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}\right)\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}\left(\frac{u^{shock}_{jk,2}}{u^{ns}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{-\beta/\nu}\right]^{\nu}$$

We can rewrite this as:

$$\frac{u^{shock}_{js2}}{u^{ns}_{js1}} = \frac{(\omega^{shock}_{js2}/P^{shock}_{j2})}{(\omega^{ns}_{js1}/P^{ns}_{j1})}$$
$$\left[\sum_{k=0}^{S}\left(\frac{\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_{k=0}^{S}\left(\frac{u^{ns}_{jk,2}}{u^{shock}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}\right)(u^{shock}_{jk,3})^{\beta/\nu}\left(\frac{u^{shock}_{jk,2}}{u^{ns}_{jk,2}}\right)^{\beta/\nu}(u^{shock}_{jk,2})^{-\beta/\nu}\right]^{\nu}$$

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{ns}/P_{j1}^{ns})} \left[\sum_{k=0}^{S} \mu_{jsk,1} (u_{jk,3}^{shock})^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} (u_{jk,2}^{shock})^{-\beta/\nu} \right]^{\nu}$$

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{ns}/P_{j1}^{ns})} \left[\sum_{k=0}^{S} \mu_{jsk,1} (u_{jk,3}^{shock})^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} (u_{jk,2}^{shock})^{-\beta/\nu} \right]^{\nu}$$

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{ns}/P_{j1}^{ns})} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\frac{u_{jk,3}^{shock}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} \right]^{\nu}$$

Remember that period 1 there was no shock, so prices are the same. Then:

$$\frac{u_{js2}^{shock}}{u_{js1}^{ns}} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{shock}/P_{j1}^{shock})} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\frac{u_{jk,3}^{shock}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} \right]^{\nu}$$

Similarly, we now that the expected utility in period one is the same:

$$\frac{u_{js2}^{shock}}{u_{js1}^{shock}} = \dot{u}_{js,2}^{shock} = \frac{(\omega_{js2}^{shock}/P_{j2}^{shock})}{(\omega_{js1}^{shock}/P_{j1}^{shock})} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\frac{u_{jk,3}^{shock}}{u_{jk,2}^{shock}} \right)^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} \right]^{\nu}$$

$$\dot{u}_{js,2}^{shock} = \frac{\dot{\omega}_{js2}^{shock}}{\dot{P}_{j2}^{shock}} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\dot{u}_{jk,3}^{shock} \right)^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,2}^{ns}} \right)^{\beta/\nu} \right]^{\nu}$$

Finally, note that in the no shock equilibrium $u_{jk,t} = u_{jk,t-1}$ for any t. Then:

$$\dot{u}_{js,2}^{shock} = \frac{\dot{\omega}_{js2}^{shock}}{\dot{P}_{j2}^{shock}} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\dot{u}_{jk,3}^{shock} \right)^{\beta/\nu} \left(\frac{u_{jk,2}^{shock}}{u_{jk,1}^{ns}} \right)^{\beta/\nu} \right]^{\nu}$$

Finally, we use the same equivalence as before and we have:

$$\dot{u}_{js,2}^{shock} = \frac{\dot{\omega}_{js2}^{shock}}{\dot{P}_{j2}^{shock}} \left[\sum_{k=0}^{S} \mu_{jsk,1} \left(\dot{u}_{jk,3}^{shock} \right)^{\beta/\nu} \left(\dot{u}_{jk,2}^{shock} \right)^{\beta/\nu} \right]^{\nu}$$

Then, period 2 dot utility in dots is slightly different than any other period. Because we have to take into account the fact that the life time utility in the first period of the shock (period 2) is different. This is exactly the same result as in CDP but in their case they have $\hat{u}_{ij,1}$ (initial period is 0). Given that the baseline economy there is no shock, we have the exact same expression. Now we need to rewrite the expression for the migration rates in period 2. Remember that:

$$\mu_{jsk,1} = \frac{u_{jk,2}^{ns}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_h u_{jh,2}^{ns}(e^{\kappa_{jsh}})^{-1/\nu}}$$

Using the fact that under no shock, all value functions are equal, and that in period 1 (steady state), nothing was predicted, and so there is no difference between shock and no-shock:

$$\mu_{jsk,1} = \frac{(u_{jk,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_h (u_{jh,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}$$

In period 2 (first shock):

$$\mu_{jsk,2} = \frac{(u_{jk,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_h (u_{jh,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}$$

And so we have:

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(u_{jk,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}{\sum_h (u_{jh,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}} \frac{\sum_h (u_{jh,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}{(u_{jk,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsk}})^{-1/\nu}}$$

Multiplying and dividing by life time utility in period 2:

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}_{jk,2}^{shock})^{\beta/\nu}(\dot{u}_{jk,3}^{shock})^{\beta/\nu}}{\sum_h (u_{jh,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}} \sum_h (u_{jh,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}$$

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}_{jk,2}^{shock})^{\beta/\nu}\dot{u}_{jk,3}^{shock}}{\sum_h \frac{(u_{jh,3}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}{\sum_h (u_{jh,1}^{shock})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}}$$

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}^{shock}_{jk,2})^{\beta/\nu}(\dot{u}^{shock}_{jk,3})^{\beta/\nu}}{\sum_h \frac{(u^{shock}_{jh,3})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}{\sum_h (u^{shock}_{jh,1})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}}$$

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}^{shock}_{jk,2})^{\beta/\nu}(\dot{u}^{shock}_{jk,3})^{\beta/\nu}}{\sum_h \left(\frac{(u^{shock}_{jh,1})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}}{\sum_h (u^{shock}_{jh,1})^{\beta/\nu}(e^{\kappa_{jsh}})^{-1/\nu}} \left(\frac{u^{shock}_{jh,3}}{u^{shock}_{jh,1}} \right)^{\beta/\nu} \right)}$$

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}^{shock}_{jk,2})^{\beta/\nu}(\dot{u}^{shock}_{jk,3})^{\beta/\nu}}{\sum_h \left(\mu_{jsh,1} \left(\frac{u^{shock}_{jh,3}}{u^{shock}_{jh,1}} \right)^{\beta/\nu} \right)}$$

Using the same trick as before:

$$\frac{\mu_{jsk,2}}{\mu_{jsk,1}} = \frac{(\dot{u}^{shock}_{jk,2})^{\beta/\nu}(\dot{u}^{shock}_{jk,3})^{\beta/\nu}}{\sum_h \mu_{jsh,1} \left(\dot{u}^{shock}_{jh,2} \right)^{\beta/\nu} \left(\dot{u}^{shock}_{jh,3} \right)^{\beta/\nu}}$$

D Welfare

We know that the life-utility in period t is:

$$V_{jst} = ln(\omega_{jst}) + ln \left(\sum_k exp \left(\beta V_{j,k,t+1} - \kappa_{jsk} \right)^{1/\nu} \right)^{\nu} + \gamma \nu_g$$

Also:

$$\mu_{j,sk,t} = \frac{\exp\left(\beta V^{shock}_{j,k,t+1} - \kappa_{j,sk,g}\right)^{1/\nu_g}}{\sum_{h=0}^{S} \exp\left(\beta V^{shock}_{j,g,h,t+1} - \kappa_{j,sh,g}\right)^{1/\nu_g}}$$

And migration frictions are 0 when s=k. Then:

$$V_{jst} = ln(\omega_{jst}) + \beta V_{js,t+1} - \nu \, ln(\mu_{jss,t}) + \gamma \nu$$

Iteration forward we have:

$$V_{jst} = \sum_{r=t}^{\infty} \beta^{r-t} \left[ln(\omega_{jsr}) - \nu ln(\mu_{jss,r}) \right] + \frac{\gamma\nu}{1-\beta}$$

Lets define the no shock value at the initial period as:

$$V_{js}^{no-shock} = \sum_{r=0}^{\infty} \beta^{r-t} \left[ln(\omega_{js}) - \nu ln(\mu_{jss}) \right] + \frac{\gamma\nu}{1-\beta}$$

And the shock value at the initial period as:

$$V_{js0}^{shock} = \sum_{r=0}^{\infty} \beta^{r} \left[ln(\omega_{jsr}) - \nu ln(\mu_{jss,r}) \right] + \frac{\gamma\nu}{1-\beta}$$

Lets define the consumption equivalent variation:

$$\frac{ln(\Delta^w)}{1-\beta} = V_{js0}^{shock} - V_{js}^{no-shock}$$

$$ln(\Delta^w) = (1-\beta) \left[\sum_{t=0}^{\infty} \beta^t \left[ln\left(\frac{\omega_{jst}}{\omega_{js}}\right) - \nu ln\left(\frac{\mu_{jss,r}}{\mu_{jss}}\right) \right] \right]$$

$$ln(\Delta^w) = \left[\sum_{t=0}^{\infty} \beta^t \left[ln\left(\frac{\omega_{jst}}{\omega_{js}}\right) - \nu ln\left(\frac{\mu_{jss,t}}{\mu_{jss}}\right) \right] \right] - \left[\sum_{t=0}^{\infty} \beta^{t+1} \left[ln\left(\frac{\omega_{jst}}{\omega_{js}}\right) - \nu ln\left(\frac{\mu_{jss,t}}{\mu_{jss}}\right) \right] \right]$$

$$
\begin{aligned}
\ln(\Delta^w) &= \left[\sum_{t=0}^{\infty}\beta^t\left(\ln\left(\frac{\omega_{jst}}{\omega_{js}}\right)-\nu\ln\left(\frac{\mu_{jss,t}}{\mu_{jss}}\right)\right)\right]-\left[\sum_{t=0}^{\infty}\beta^{t+1}\left(\ln\left(\frac{\omega_{jst}}{\omega_{js}}\right)-\nu\ln\left(\frac{\mu_{jss,t}}{\mu_{jss}}\right)\right)\right] \\
&\equiv \sum_{t=0}^{\infty}\beta^t x_t-\sum_{t=0}^{\infty}\beta^{t+1}x_t, \qquad x_t\equiv\ln\left(\frac{\omega_{jst}}{\omega_{js}}\right)-\nu\ln\left(\frac{\mu_{jss,t}}{\mu_{jss}}\right) \\
&= \sum_{t=0}^{\infty}\beta^t x_t-\sum_{t=1}^{\infty}\beta^t x_{t-1} \qquad \text{(reindex } u=t+1 \text{ in the second sum)} \\
&= x_0+\sum_{t=1}^{\infty}\beta^t x_t-\sum_{t=1}^{\infty}\beta^t x_{t-1} \\
&= x_0+\sum_{t=1}^{\infty}\beta^t\left(x_t-x_{t-1}\right) \\
&= x_0+\sum_{t=1}^{\infty}\beta^t\left[\ln\left(\frac{\omega_{jst}}{\omega_{js,t-1}}\right)-\nu\ln\left(\frac{\mu_{jss,t}}{\mu_{jss,t-1}}\right)\right] \\
&= x_0+\sum_{t=1}^{\infty}\beta^t\left[\ln\left(\dot{\omega}_{jst}\right)-\nu\ln\left(\dot{\mu}_{jss,t}\right)\right], \qquad \dot{\omega}_{jst}\equiv\frac{\omega_{jst}}{\omega_{js,t-1}},\ \dot{\mu}_{jss,t}\equiv\frac{\mu_{jss,t}}{\mu_{jss,t-1}}.
\end{aligned}
$$

Given that in the initial period there is no change in real wage, we have:

$$
\ln(\Delta^w)=\sum_{t=1}^{\infty}\beta^t\left[\ln\left(\dot{\omega}_{jst}\right)-\nu\ln\left(\dot{\mu}_{jss,t}\right)\right].
$$

E Elasticity of substitution

The definition of the aggregate elasticity of substitution is:

$$
\varepsilon_{is}^{agg} \equiv \frac{d\log\left(\frac{L_{iHs}}{L_{iLs}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.
$$

For any firm type $p \in \{hp, lp\}$,

$$\frac{L^p_{iHs}}{L^p_{iLs}} = \frac{\mu_{is,p}}{1-\mu_{is,p}} \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho}.$$

Equivalently,

$$L^p_{iHs} = \frac{\mu_{is,p}}{1-\mu_{is,p}} \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho} L^p_{iLs}.$$

Summing over $p \in \{hp, lp\}$ yields

$$L_{iHs} = L^{hp}_{iHs} + L^{lp}_{iHs} = \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho} \left[\frac{\mu^{hp}_{is}}{1-\mu^{hp}_{is}} L^{hp}_{iLs} + \frac{\mu^{lp}_{is}}{1-\mu^{lp}_{is}} L^{lp}_{iLs}\right].$$

Using $L_{iLs} = L^{hp}_{iLs} + L^{lp}_{iLs}$, define $s^{hp}_{is} \equiv \frac{L^{hp}_{iLs}}{L_{iLs}}$. Then dividing by L_{iLs} to obtain:

$$\frac{L_{iHs}}{L_{iLs}} = \left(\frac{W_{iLs}}{W_{iHs}}\right)^{\rho} \left[s^{hp}_{is} \frac{\mu^{hp}_{is}}{1-\mu^{hp}_{is}} + (1-s^{hp}_{is}) \frac{\mu^{lp}_{is}}{1-\mu^{lp}_{is}}\right].$$

Taking logs gives:

$$\log\left(\frac{L_{iHs}}{L_{iLs}}\right) = \rho \log\left(\frac{W_{iLs}}{W_{iHs}}\right) + \log\left[s^{hp}_{is} \frac{\mu^{hp}_{is}}{1-\mu^{hp}_{is}} + (1-s^{hp}_{is}) \frac{\mu^{lp}_{is}}{1-\mu^{lp}_{is}}\right].$$

Differentiating with respect to $\log\left(\frac{W_{iLs}}{W_{iHs}}\right)$ yields

$$\boxed{\varepsilon^{agg}_{is} = \rho + \frac{d}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} \log\left[s^{hp}_{is} \frac{\mu^{hp}_{is}}{1-\mu^{hp}_{is}} + (1-s^{hp}_{is}) \frac{\mu^{lp}_{is}}{1-\mu^{lp}_{is}}\right]}.$$

Assume

$$\mu^{lp}_{is} = \kappa\, \mu^{hp}_{is}, \qquad 0 < \kappa < 1.$$

Then the bracketed term becomes

$$s_{is}^{hp}\frac{\mu_{is}^{hp}}{1-\mu_{is}^{hp}}+(1-s_{is}^{hp})\frac{\mu_{is}^{lp}}{1-\mu_{is}^{lp}}=s_{is}^{hp}\frac{\mu_{is}^{hp}}{1-\mu_{is}^{hp}}+(1-s_{is}^{hp})\frac{\kappa\mu_{is}^{hp}}{1-\kappa\mu_{is}^{hp}}=\mu_{is}^{hp}\left[\frac{s_{is}^{hp}}{1-\mu_{is}^{hp}}+\frac{\kappa(1-s_{is}^{hp})}{1-\kappa\mu_{is}^{hp}}\right]$$

And so we have:

$$\begin{aligned}\varepsilon_{is}^{agg}&=\rho+\frac{d}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}\log\left(\mu_{is}^{hp}\left[\frac{s_{is}^{hp}}{1-\mu_{is}^{hp}}+\frac{\kappa(1-s_{is}^{hp})}{1-\kappa\mu_{is}^{hp}}\right]\right)\\&=\rho+\frac{d}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}\log\left[\frac{s_{is}^{hp}}{1-\mu_{is}^{hp}}+\frac{\kappa(1-s_{is}^{hp})}{1-\kappa\mu_{is}^{hp}}\right],\end{aligned}$$

Define the constants (only parameters):

$$A_{is}\equiv\frac{1}{1-\mu_{is}^{hp}},\qquad B_{is}\equiv\frac{\kappa}{1-\kappa\mu_{is}^{hp}}$$

Then

$$\left[\frac{s_{is}^{hp}}{1-\mu_{is}^{hp}}+\frac{\kappa(1-s_{is}^{hp})}{1-\kappa\mu_{is}^{hp}}\right]=A_{is}\,s_{is}^{hp}+B_{is}(1-s_{is}^{hp})=B_{is}+(A_{is}-B_{is})\,s_{is}^{hp}.$$

Hence

$$\varepsilon_{is}^{agg}=\rho+\frac{d}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}\log\left[B_{is}+(A_{is}-B_{is})\,s_{is}^{hp}\right].$$

$$\varepsilon_{is}^{agg}=\rho+\frac{(A_{is}-B_{is})}{B_{is}+(A_{is}-B_{is})s_{is}^{hp}}\cdot\frac{ds_{is}^{hp}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

From (6), define the (type-specific) unskilled-wage-bill terms:

$$W_{iLs}L_{iLs}^{hp} \equiv (1-\mu_{is}^{hp})\left(\frac{W_{is}^{hp}}{W_{iLs}}\right)^{\rho-1}\alpha_{is}R_{is}^{hp},$$
$$W_{iLs}L_{iLs}^{lp} \equiv (1-\mu_{is}^{lp})\left(\frac{W_{is}^{lp}}{W_{iLs}}\right)^{\rho-1}\alpha_{is}R_{is}^{lp}.$$

Then $W_{iLs}L_{iLs} = W_{iLs}L_{iLs}^{hp} + W_{iLs}L_{iLs}^{lp}$ holds identically. By definition,

$$s_{is}^{hp} \equiv \frac{L_{iLs}^{hp}}{L_{iLs}} = \frac{W_{iLs}L_{iLs}^{hp}}{W_{iLs}L_{iLs}} = \frac{W_{iLs}L_{iLs}^{hp}}{W_{iLs}L_{iLs}^{hp} + W_{iLs}L_{iLs}^{lp}}.$$

Combining the last expressions:

$$s_{is}^{hp} = \frac{(1-\mu_{is}^{hp})\left(\frac{W_{is}^{hp}}{W_{iLs}}\right)^{\rho-1}R_{is}^{hp}}{(1-\mu_{is}^{hp})\left(\frac{W_{is}^{hp}}{W_{iLs}}\right)^{\rho-1}R_{is}^{hp} + (1-\mu_{is}^{lp})\left(\frac{W_{is}^{lp}}{W_{iLs}}\right)^{\rho-1}R_{is}^{lp}}.$$

Define the odds ratio

$$\Omega_{is} \equiv \frac{s_{is}^{hp}}{1-s_{is}^{hp}} = \frac{(1-\mu_{is}^{hp})\left(\frac{W_{is}^{hp}}{W_{iLs}}\right)^{\rho-1}R_{is}^{hp}}{(1-\mu_{is}^{lp})\left(\frac{W_{is}^{lp}}{W_{iLs}}\right)^{\rho-1}R_{is}^{lp}}.$$

Then

$$\frac{ds_{is}^{hp}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = s_{is}^{hp}(1-s_{is}^{hp})\cdot\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

And so we have:

$$\varepsilon_{is}^{agg} = \rho + \underbrace{\frac{(A_{is}-B_{is})}{B_{is}+(A_{is}-B_{is})s_{is}^{hp}}\cdot s_{is}^{hp}(1-s_{is}^{hp})}_{>0}\cdot\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

Where:

$$log\,\Omega_{is} = log\left(\frac{(1-\mu_{is})}{(1-\kappa\mu_{is})}\right) - (\rho-1)[\log(W_{is}^{lp}) - log(W_{is}^{hp})] - [log(R_{is}^{lp}) - log(R_{is}^{hp})]$$

Then, the derivative is:

$$\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = (\rho-1)\frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} + \frac{d\log\left(\frac{R_{is}^{hp}}{R_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

Remember that:

$$W_{is}^{p} = \left[\mu_{is}^{p}\, W_{iHs}^{1-\rho} + (1-\mu_{is}^{p})\, W_{iLs}^{1-\rho}\right]^{\frac{1}{1-\rho}}, \qquad p \in \{hp, lp\}.$$

$$\theta_{L,is}^{p} \equiv \frac{\partial \log W_{is}^{p}}{\partial \log W_{iLs}} = \frac{(1-\mu_{is}^{p})\, W_{iLs}^{1-\rho}}{\mu_{is}^{p} W_{iHs}^{1-\rho} + (1-\mu_{is}^{p}) W_{iLs}^{1-\rho}} \in (0,1).$$

Assume W_{iHs} fixed, then $d\log\left(\frac{W_{iLs}}{W_{iHs}}\right) = d\log W_{iLs}$, and so:

$$\frac{d\log W_{is}^{p}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \theta_{L,is}^{p}, \qquad \frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \theta_{L,is}^{hp} - \theta_{L,is}^{lp}.$$

Without mark-ups and with a common total labor share, all the other intermediate prices that influence the price of the firm are equal, and so we have:

$$R_{is}^{lp} = \lambda_{iis} E_{is} = \frac{(p_{iis}^{lp})^{1-\sigma_s}}{P_{is}^{1-\sigma_s}} E_{is} = \left(\frac{1}{T_{is}^{\text{lp}}}\left(W_{is}^{\text{lp}}\right)^{\alpha_{is}} \prod_{k} P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{is}}{P_{is}^{1-\sigma_s}}$$

$$R_{is}^{hp} = \sum_{j} \lambda_{ijs} E_{ijs} = \sum_{j} \frac{(p_{ijs}^{hp})^{1-\sigma_s}}{P_{is}^{1-\sigma_s}} E_{js} = \sum_{j} \left(\frac{\tau_{ijs}}{T_{is}^{\text{hp}}}\left(W_{is}^{\text{hp}}\right)^{\alpha_{is}} \prod_{k} P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{js}}{P_{js}^{1-\sigma_s}}$$

Then note that:

$$\frac{R_{is}^{lp}}{R_{is}^{hp}} = \left(\frac{W_{is}^{lp}}{W_{is}^{hp}}\right)^{\alpha_{is}(1-\sigma_s)} \frac{\left[\left(\frac{1}{T_{is}^{\text{lp}}}\prod_k P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{is}}{P_{is}^{1-\sigma_s}}\right]}{\left[\sum_j \left(\frac{\tau_{ijs}}{T_{is}^{\text{lp}}}\prod_k P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{js}}{P_{js}^{1-\sigma_s}}\right]}$$

Calling the last ratio the *GE effects*, define

$$\mathcal{G}_{is} \equiv \frac{\left[\left(\frac{1}{T_{is}^{lp}}\prod_k P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{is}}{P_{is}^{1-\sigma_s}}\right]}{\left[\sum_j \left(\frac{\tau_{ijs}}{T_{is}^{hp}}\prod_k P_{ik}^{\alpha_{iks}}\right)^{1-\sigma_s} \frac{E_{js}}{P_{js}^{1-\sigma_s}}\right]}.$$

Then your revenue ratio can be written compactly as

$$\frac{R_{is}^{lp}}{R_{is}^{hp}} = \left(\frac{W_{is}^{lp}}{W_{is}^{hp}}\right)^{\alpha_{is}(1-\sigma_s)} \mathcal{G}_{is}, \qquad \Rightarrow \qquad \frac{R_{is}^{hp}}{R_{is}^{lp}} = \left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)^{\alpha_{is}(1-\sigma_s)} \mathcal{G}_{is}^{-1}.$$

Taking logs,

$$\log\left(\frac{R_{is}^{hp}}{R_{is}^{lp}}\right) = \alpha_{is}(1-\sigma_s)\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right) - \log\mathcal{G}_{is}.$$

Differentiating w.r.t. $\log\left(\frac{W_{iLs}}{W_{iHs}}\right)$ yields

$$\frac{d\log\left(\frac{R_{is}^{hp}}{R_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \alpha_{is}(1-\sigma_s)\frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} - \frac{d\log\mathcal{G}_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

Substituting this into

$$\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = (\rho-1)\frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} + \frac{d\log\left(\frac{R_{is}^{hp}}{R_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)},$$

we obtain

$$\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \Big[(\rho-1)+\alpha_{is}(1-\sigma_s)\Big]\frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} - \frac{d\log\mathcal{G}_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

Using

$$\frac{d\log\left(\frac{W_{is}^{hp}}{W_{is}^{lp}}\right)}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \theta_{L,is}^{hp} - \theta_{L,is}^{lp},$$

this becomes

$$\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)} = \Big[(\rho-1)+\alpha_{is}(1-\sigma_s)\Big]\left(\theta_{L,is}^{hp} - \theta_{L,is}^{lp}\right) - \frac{d\log\mathcal{G}_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}.$$

Finally, substituting into the aggregate elasticity expression

$$\varepsilon_{is}^{agg} = \rho + \underbrace{\frac{(A_{is}-B_{is})}{B_{is}+(A_{is}-B_{is})s_{is}^{hp}}\cdot s_{is}^{hp}(1-s_{is}^{hp})}_{>0}\cdot\frac{d\log\Omega_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)},$$

we obtain

$$\varepsilon_{is}^{agg} = \rho + \underbrace{\frac{(A_{is}-B_{is})}{B_{is}+(A_{is}-B_{is})s_{is}^{hp}}\cdot s_{is}^{hp}(1-s_{is}^{hp})}_{>0} \times\left\{\Big[(\rho-1)+\alpha_{is}(1-\sigma_s)\Big]\left(\theta_{L,is}^{hp}-\theta_{L,is}^{lp}\right) - \frac{d\log\mathcal{G}_{is}}{d\log\left(\frac{W_{iLs}}{W_{iHs}}\right)}\right\}.$$

Assuming that the trade-elasticity term dominates the within-firm substitution term, i.e.

$$\alpha_{is}(\sigma_s - 1) > (\rho - 1) \qquad \Longleftrightarrow \qquad (\rho - 1) + \alpha_{is}(1 - \sigma_s) < 0,$$

and recalling that low-productivity firms are more unskilled-intensive,

$$\theta^{lp}_{L,is} > \theta^{hp}_{L,is} \qquad \Longrightarrow \qquad \theta^{hp}_{L,is} - \theta^{lp}_{L,is} < 0,$$

it follows (ignoring the GE term) that

$$\frac{d \log \Omega_{is}}{d \log\left(\frac{W_{iLs}}{W_{iHs}}\right)} \approx \Big[(\rho - 1) + \alpha_{is}(1 - \sigma_s)\Big] \left(\theta^{hp}_{L,is} - \theta^{lp}_{L,is}\right) > 0.$$

And so the aggregate elasticity of substitution between types of workers is higher than ρ: $\epsilon^{agg}_{is} > \rho$.

$$\varepsilon^{agg}_{is} \approx \rho + \underbrace{\frac{(A_{is} - B_{is})}{B_{is} + (A_{is} - B_{is}) s^{hp}_{is}} \cdot s^{hp}_{is}(1 - s^{hp}_{is})}_{>0} \cdot \underbrace{\Big[(\rho - 1) + \alpha_{is}(1 - \sigma_s)\Big]}_{<0} \underbrace{\left(\theta^{hp}_{L,is} - \theta^{lp}_{L,is}\right)}_{<0}$$

For high values of κ, the second term is small.